

Online Appendix for
“The Term Structure of Currency Carry Trade Risk Premia”
—Not For Publication—

This Online Appendix describes additional empirical and theoretical results on foreign bond returns in U.S. dollars.

- Section **A** gathers robustness checks on empirical results: subsection **A.1** reports additional time-series predictability results; subsection **A.2** reports additional results on portfolios of countries sorted by the deviation of their short-term interest rate from its 10-year rolling mean; subsection **A.3** reports results on portfolios of countries sorted by the short-term interest rate level; subsection **A.4** reports additional results for portfolios of countries sorted by the slope of their yield curve; subsection **A.5** reports additional results obtained with zero-coupon bonds.
- Section **B** compares finite to infinite maturity bond returns in the benchmark Joslin, Singleton, and Zhu (2011) term structure model.
- Section **C** reports additional theoretical results on dynamic term structure models, starting with the simple Vasicek (1977) and Cox, Ingersoll, and Ross (1985) one-factor models, before turning to their k -factor extensions and the model studied in Lustig, Roussanov, and Verdelhan (2014).
- Section **D** studies the incomplete market version of the benchmark Cox, Ingersoll, and Ross (1985) model.

A Robustness Checks on Time-Series and Cross-Sectional Tests

A.1 Individual Country Time-Series Predictability Results

We check the robustness of our time-series predictability results by considering different sample windows and different holding periods. Tables 6 and 7 report the output of three-month return predictability regressions for bond and currency excess returns over both a long sample (1/1951–12/2015) and our benchmark sample (1/1975–12/2015), as well as for zero-coupon bonds for our benchmark sample (1/1975–12/2015). While we obtain some dollar bond excess return predictability using yield curve slope differentials on one-month returns obtained with bond indices in the post-Bretton Woods sample (reported in the main text), we do not find any such predictability for three-month returns on zero-coupon bonds (Panel C of Table 7), thus reinforcing our broad findings.

A.2 Portfolio Cross-Sectional Evidence: Sorting by Interest Rate Deviations

We now turn our attention to the robustness of our cross-sectional results by considering different samples of countries, different time windows, and different holding periods. We start with currency portfolios sorted on deviations of interest rates from their 10-year rolling mean.

A.2.1 Benchmark Sample

Figure 4 plots the composition of the three portfolios of the currencies of the benchmark sample sorted on interest rate deviations, ranked from low (Portfolio 1) to high (Portfolio 3).

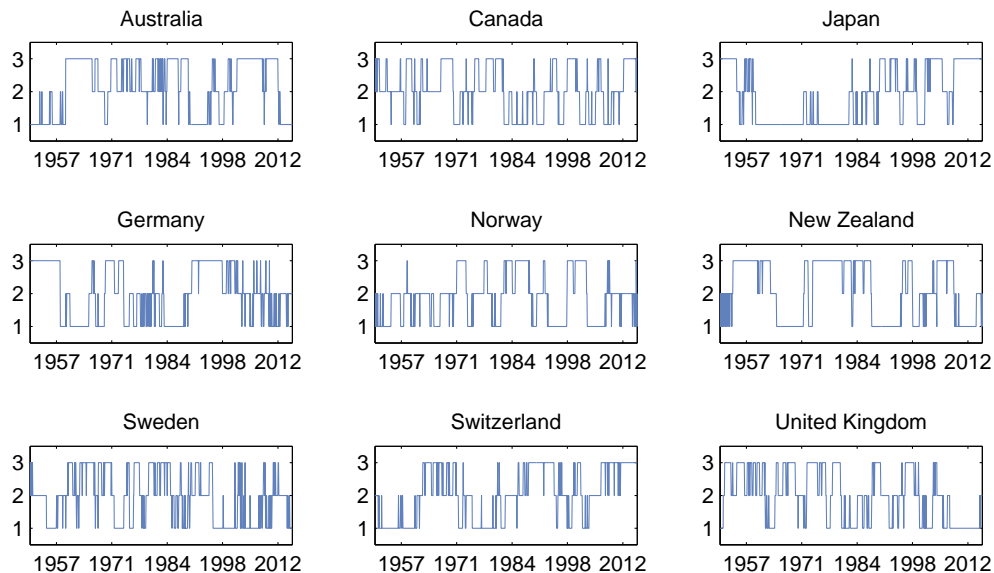


Figure 4: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 9 currencies sorted by the deviation of their short-term interest rates from the corresponding 10-year rolling mean. The portfolios are rebalanced monthly. Data are monthly, from 1/1951 to 12/2015.

Figure 5 presents the cumulative one-month log excess returns on investments in foreign Treasury bills and foreign 10-year bonds. Over the entire 1/1951 – 12/2015 sample period, the average currency log excess return of the carry trade strategy (long Portfolio 3, short Portfolio 1) is 2.52% per year, whereas the local currency bond log excess return is -3.81% per year. Thus, the interest rate carry trade implemented using 10-year bonds yields an average annualized dollar return of -1.29% , which is not statistically significant (bootstrap standard error of 0.94%). The average inflation rate of Portfolio 1 is 3.56% and its average credit rating is 1.44 (1.51 when adjusted for outlook), while the average inflation rate of Portfolio 3 is 4.72% and its average credit rating

Table 6: Return Predictability of Interest Rate Differentials: Three-month Holding Period

	Bond dollar return difference			Currency excess return			Bond local currency return difference			Obs.
	β	s.e.	$R^2(\%)$	β	s.e.	$R^2(\%)$	β	s.e.	$R^2(\%)$	
Section I: coupon bonds										
Panel A: 1/1951–12/2015										
Australia	0.35	[0.61]	0.03	0.86**	[0.43]	1.38	-0.51	[0.38]	0.79	778
Canada	-0.41	[0.41]	0.06	0.95***	[0.33]	1.55	-1.36***	[0.28]	5.25	778
Germany	1.43	[0.89]	1.15	1.60**	[0.73]	1.88	-0.18	[0.48]	-0.07	778
Japan	1.36***	[0.39]	3.52	1.29***	[0.27]	5.39	0.07	[0.28]	-0.11	778
New Zealand	0.64	[0.56]	0.44	1.29***	[0.39]	4.91	-0.66*	[0.38]	1.18	778
Norway	0.63	[0.48]	0.47	1.17**	[0.47]	3.44	-0.55*	[0.31]	0.74	778
Sweden	-0.61	[0.79]	0.33	0.26	[0.84]	-0.01	-0.87**	[0.40]	1.84	778
Switzerland	1.31*	[0.69]	1.84	1.76***	[0.62]	3.97	-0.45	[0.39]	0.50	778
United Kingdom	1.40	[0.95]	1.13	1.86**	[0.76]	3.65	-0.46	[0.53]	0.21	778
Panel	0.73**	[0.32]	0.67	1.20***	[0.30]	2.84	-0.47**	[0.19]	0.64	7002
Panel B: 1/1975–12/2015										
Australia	0.94	[0.81]	0.56	1.37***	[0.52]	2.23	-0.43	[0.57]	0.29	490
Canada	-0.37	[0.56]	-0.08	1.21***	[0.45]	1.91	-1.57***	[0.33]	7.31	490
Germany	1.34	[1.05]	1.19	1.77**	[0.88]	2.57	-0.43	[0.56]	0.21	490
Japan	2.48***	[0.76]	4.09	2.71***	[0.61]	7.06	-0.22	[0.51]	-0.11	490
New Zealand	1.26*	[0.74]	1.22	1.94***	[0.48]	6.94	-0.68	[0.56]	0.71	490
Norway	1.02*	[0.57]	1.36	1.51***	[0.54]	4.69	-0.49	[0.38]	0.64	490
Sweden	-0.46	[0.90]	0.04	0.33	[0.98]	-0.04	-0.78*	[0.45]	1.47	490
Switzerland	1.36*	[0.76]	2.05	1.99***	[0.70]	4.82	-0.63	[0.41]	1.09	490
United Kingdom	1.78	[1.10]	1.71	2.06**	[0.90]	3.86	-0.29	[0.62]	-0.07	490
Panel	1.06**	[0.46]	1.06	1.63***	[0.43]	3.68	-0.57**	[0.28]	0.81	4410
Section II: Zero-Coupon Bonds, 1/1975–12/2015										
Australia	2.17*	[1.25]	2.88	1.45*	[0.82]	1.55	0.72	[0.88]	0.54	344
Canada	0.49	[0.78]	-0.12	1.47***	[0.51]	2.15	-0.98*	[0.56]	1.56	357
Germany	1.53*	[0.91]	1.36	1.72**	[0.82]	2.42	-0.19	[0.59]	-0.16	490
Japan	1.88*	[1.00]	1.61	2.66***	[0.93]	4.73	-0.78	[0.57]	0.48	369
New Zealand	1.31	[1.98]	0.18	0.28	[1.93]	-0.30	1.03	[1.17]	0.49	309
Norway	1.90	[1.74]	1.22	0.43	[1.75]	-0.36	1.47	[1.00]	2.05	213
Sweden	1.95	[1.28]	1.82	1.52	[1.13]	1.33	0.43	[0.92]	-0.13	274
Switzerland	1.91*	[0.98]	2.42	2.51**	[1.13]	4.59	-0.60	[0.76]	0.29	333
United Kingdom	2.28*	[1.30]	2.36	1.84*	[1.02]	3.18	0.45	[0.80]	-0.04	441
Panel	1.81***	[0.56]	1.84	1.72***	[0.57]	2.32	0.08	[0.34]	0.01	3130

Notes: The table reports regression results obtained when regressing the bond dollar return difference, defined as the difference between the log return on foreign bonds (expressed in U.S. dollars) and the log return of U.S. bonds in U.S. dollars, or the currency excess return, defined as the difference between the log return on foreign Treasury bills (expressed in U.S. dollars) and the log return of U.S. Treasury bills in U.S. dollars, or the bond local currency return difference, defined as the difference between the log return on foreign bonds (expressed in local currency terms) and the log return of U.S. bonds in U.S. dollars, on the corresponding interest rate differential, defined as the difference between the foreign nominal interest rate and the U.S. nominal interest rate. Section I uses coupon bonds, whereas Section II uses zero-coupon bonds. The holding period is three months and returns are sampled monthly. The log returns and the interest rate differentials are annualized. In Panel A of Section I, the sample period is 1/1951–12/2015, whereas in Panel B of Section I the sample period is 1/1975–12/2015. In Section II, the sample period is 1/1975–12/2015. In individual country regressions, standard errors are obtained with a Newey-West approximation of the spectral density matrix with four lags. Panel regressions include country fixed effects, and standard errors are obtained using the Driscoll and Kraay (1998) methodology with four lags. One, two, and three stars denote statistical significance at the 10%, 5%, and 1% level, respectively.

Table 7: Return Predictability of Yield Curve Slope Differentials: Three-month Holding Period

	Bond dollar return difference			Currency excess return			Bond local currency return difference			Obs.
	β	s.e.	$R^2(\%)$	β	s.e.	$R^2(\%)$	β	s.e.	$R^2(\%)$	
Section I: coupon bonds										
Panel A: 1/1951–12/2015										
Australia	0.78	[0.79]	0.27	-0.91	[0.62]	0.72	1.69***	[0.45]	4.91	778
Canada	1.53***	[0.51]	1.86	-0.70*	[0.39]	0.58	2.23***	[0.33]	11.07	778
Germany	1.25	[0.85]	0.66	-0.57	[0.69]	0.07	1.82***	[0.49]	5.35	778
Japan	0.02	[0.78]	-0.13	-1.99***	[0.52]	3.64	2.01***	[0.57]	4.92	778
New Zealand	1.32	[1.20]	1.13	-0.84	[0.70]	0.95	2.16***	[0.72]	7.08	778
Norway	-0.04	[0.75]	-0.13	-1.37*	[0.73]	2.20	1.33***	[0.43]	2.32	778
Sweden	2.48**	[1.01]	4.54	0.53	[1.02]	0.18	1.95***	[0.54]	5.96	778
Switzerland	0.66	[0.89]	0.07	-2.03**	[0.91]	2.10	2.69***	[0.46]	8.95	778
United Kingdom	0.52	[1.02]	0.01	-1.55*	[0.86]	1.95	2.07***	[0.49]	5.36	778
Panel	0.96*	[0.56]	0.58	-0.99**	[0.49]	0.98	1.96***	[0.31]	5.70	7002
Panel B: 1/1975–12/2015										
Australia	0.71	[1.27]	-0.03	-1.52	[0.95]	1.01	2.24***	[0.77]	5.18	490
Canada	2.18***	[0.76]	2.34	-0.99*	[0.56]	0.62	3.17***	[0.43]	17.47	490
Germany	0.15	[1.50]	-0.20	-2.06*	[1.15]	1.55	2.21**	[0.87]	4.99	490
Japan	-1.25	[1.18]	0.31	-3.82***	[0.95]	6.57	2.56***	[0.74]	5.75	490
New Zealand	2.10	[2.01]	1.69	-1.11	[1.11]	0.91	3.21***	[1.14]	9.72	490
Norway	-0.48	[0.90]	-0.05	-1.80**	[0.85]	2.96	1.32**	[0.53]	2.60	490
Sweden	2.73**	[1.20]	4.64	0.73	[1.25]	0.25	2.01***	[0.67]	5.85	490
Switzerland	0.04	[0.99]	-0.20	-2.76***	[1.03]	3.49	2.80***	[0.51]	9.66	490
United Kingdom	0.44	[1.43]	-0.13	-2.27*	[1.22]	2.91	2.70***	[0.64]	7.40	490
Panel	0.85	[0.76]	0.33	-1.55**	[0.67]	1.59	2.41***	[0.41]	6.82	4410
Section II: Zero-Coupon Bonds, 1/1975–12/2015										
Australia	-0.28	[2.04]	-0.27	-1.49	[1.78]	0.37	1.21	[1.31]	0.52	344
Canada	1.43	[1.06]	0.43	-1.39**	[0.66]	0.86	2.82***	[0.69]	7.74	357
Germany	0.58	[0.98]	-0.06	-1.55*	[0.89]	1.16	2.12***	[0.74]	3.77	490
Japan	-1.77	[1.32]	0.60	-5.01***	[1.21]	9.02	3.24***	[0.84]	6.53	369
New Zealand	1.82	[2.28]	0.34	0.83	[2.37]	-0.16	0.98	[1.07]	0.19	309
Norway	-0.57	[1.91]	-0.37	0.29	[1.96]	-0.44	-0.86	[1.20]	0.09	213
Sweden	1.20	[1.98]	0.03	-0.51	[1.98]	-0.28	1.71	[1.20]	1.49	274
Switzerland	-0.49	[1.04]	-0.18	-2.59*	[1.32]	3.31	2.10**	[0.87]	4.82	333
United Kingdom	0.12	[1.61]	-0.22	-1.64	[1.39]	1.39	1.76*	[0.96]	1.51	441
Panel	0.10	[0.73]	0.00	-1.75**	[0.81]	1.34	1.85***	[0.46]	2.43	3130

Notes: The table reports regression results obtained when regressing the bond dollar return difference, defined as the difference between the log return on foreign bonds (expressed in U.S. dollars) and the log return of U.S. bonds in U.S. dollars, or the currency excess return, defined as the difference between the log return on foreign Treasury bills (expressed in U.S. dollars) and the log return of U.S. Treasury bills in U.S. dollars, or the bond local currency return difference, defined as the difference between the log return on foreign bonds (expressed in local currency terms) and the log return of U.S. bonds in U.S. dollars, on the corresponding yield curve slope differential, defined as the difference between the foreign nominal yield curve slope and the U.S. nominal yield curve slope. Section I uses coupon bonds, whereas Section II uses zero-coupon bonds. The holding period is three months and returns are sampled monthly. The log returns and the yield curve slope differentials are annualized. In Panel A of Section I, the sample period is 1/1951–12/2015, whereas in Panel B of Section I the sample period is 1/1975–12/2015. In Section II, the sample period is 1/1975–12/2015. In individual country regressions, standard errors are obtained with a Newey-West approximation of the spectral density matrix with four lags. Panel regressions include country fixed effects, and standard errors are obtained using the Driscoll and Kraay (1998) methodology with four lags. One, two, and three stars denote statistical significance at the 10%, 5%, and 1% level, respectively.

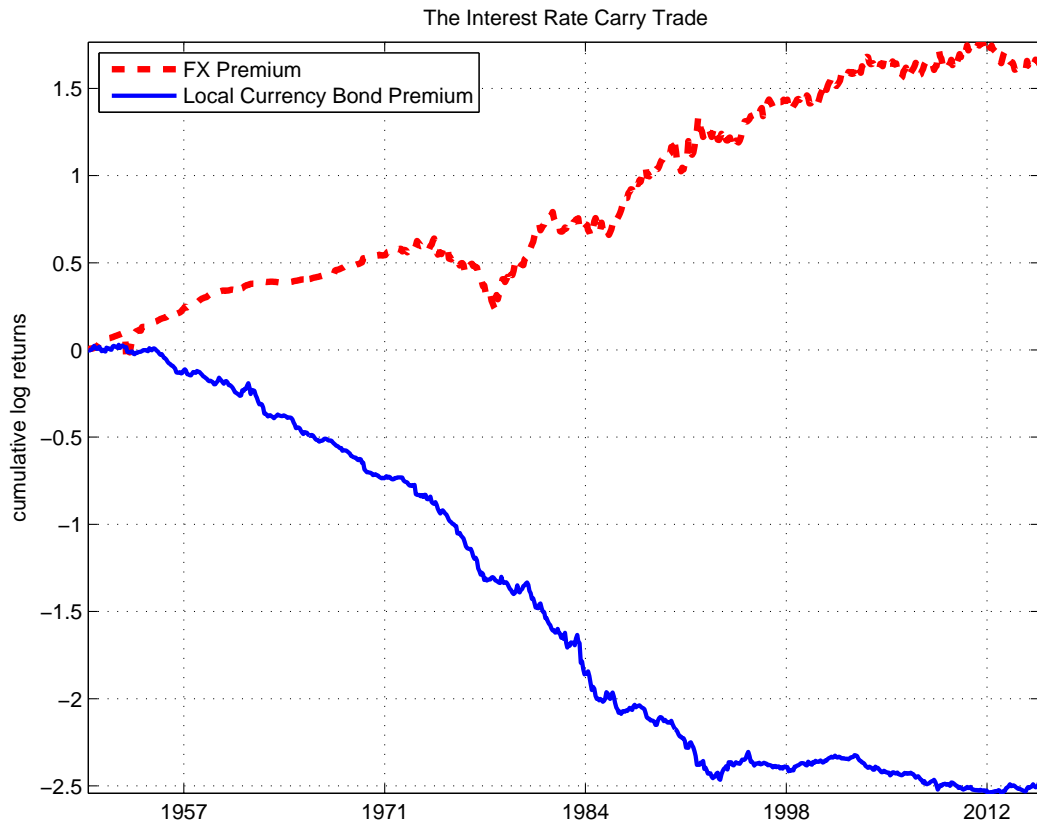


Figure 5: The Carry Trade and Term Premia – The figure presents the cumulative one-month log excess returns on investments in foreign Treasury bills and foreign 10-year bonds. The benchmark panel of countries includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. Countries are sorted every month into three portfolios by the level of the deviation of their one-month interest rate from its 10-year rolling mean. The returns correspond to a strategy going long in the Portfolio 3 and short in Portfolio 1. The sample period is 1/1951–12/2015.

is 1.46 (1.81 when adjusted for outlook). Therefore, countries with high local currency bond term premia have low inflation and high credit ratings on average, whereas countries with low term premia have high average inflation rates and low average credit ratings, which suggests that the offsetting effect of the local currency bond excess returns is not due to compensation for inflation or credit risk. As seen in Table 2, our findings are very similar when we consider only the post-Bretton Woods period (1/1975 – 12/2015). Finally, we turn to the 7/1989 – 12/2015 period. The one-month average currency excess return of the carry trade strategy is 2.33%, largely offset by the local currency bond excess return of -1.33% . As a result, the average dollar bond excess return is 1.00%, which is not statistically significant, as its bootstrap standard error is 1.47%. Portfolio 1 has an average inflation rate of 1.91% and an average credit rating of 1.67 (1.72 when adjusted for outlook), whereas Portfolio 3 has an average inflation rate of 2.05% and an average credit rating of 1.67 (1.73 when adjusted for outlook).

We find very similar results when we increase the holding period k from 1 to 3 or 12 months: there is no evidence of statistically significant differences in dollar bond premia across the currency portfolios. In particular, for the entire 1/1951 – 12/2015 period, the annualized dollar excess return of the carry trade strategy implemented using 10-year bonds is a non-significant -0.68% (bootstrap standard error of 1.12%) for the 3-month holding period, as the average currency risk premium of 2.04% is offset by the average local currency bond premium of -2.72% . For the 12-month horizon, the average currency risk premium is 1.52%, which is almost fully offset by the average local currency bond premium of -1.68% , yielding an average dollar bond premium of -0.15% (bootstrap standard error of 1.08%). The corresponding average dollar bond premium for the post-Bretton Woods sample (1/1975 – 12/2015) is

−0.88% for the 3-month holding period (average currency risk premium of 1.81%, average local currency bond premium of −2.68%) and −0.57% for the 12-month holding period (average currency risk premium of 1.28%, average local currency bond premium of −1.85%), neither of which is statistically significant (the bootstrap standard error is 1.39% and 1.55%, respectively). Finally, we consider the 7/1989 – 12/2015 period. The average dollar bond premium is 0.68% for the 3-month horizon (average currency risk premium of 1.39%, average local currency bond premium of −0.71%) and 0.86% for the 12-month horizon (average currency risk premium of 1.37%, average local currency bond premium of −0.51%). Neither of those average dollar bond premia is statistically significant, as their bootstrap standard error is 1.58% and 1.62%, respectively.

A.2.2 Developed Countries

Very similar patterns of risk premia emerge using larger sets of countries. In the sample of 20 developed countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the U.K.), we sort currencies in four portfolios, the composition of which is plotted in Figure 6.

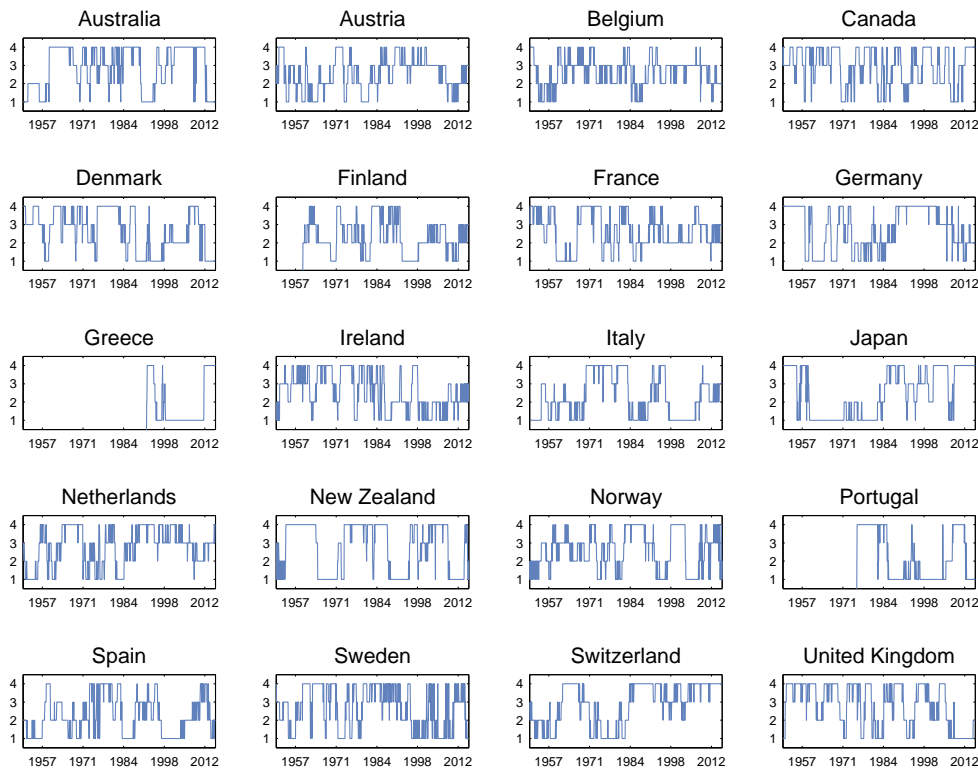


Figure 6: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 20 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 1/1951 to 12/2015.

We start with 1-month holding period returns. Over the long sample period (1/1951 – 12/2015), the average currency log excess return of the carry trade is 1.32% per year, whereas the local currency bond log excess return is −4.77% per year. Therefore, the 10-year bond carry trade strategy yields a marginally significant average annualized return of −3.45% (bootstrap standard error of 1.97%). The average inflation rate of Portfolio 1 is 4.04% and its average credit rating is 2.68 (2.58 when adjusted for outlook); in comparison, the average inflation rate of Portfolio 4 is 5.05% and its average credit rating is 2.24 (2.41 when adjusted for outlook). We find similar results when we focus on the post-Bretton Woods sample: the average currency log excess return is 1.38% per year, offset by a local currency bond log excess return of −2.85%, so the 10-year bond carry trade strategy yields a statistically not significant annualized dollar excess return of −1.47% (bootstrap standard error of 1.15%). The average inflation rate of Portfolio 1

is 3.72% and its average credit rating is 2.71 (2.64 adjusted for outlook), whereas the average inflation rate of Portfolio 4 is 5.11% and its average credit rating is 2.31 (2.49 adjusted for outlook).

We now turn to longer holding periods. For the 1/1951 – 12/2015 sample, the annualized dollar excess return of the carry trade strategy implemented using 10-year bonds is a non-significant -1.15% (bootstrap standard error of 2.02%) for the 3-month holding period and a non-significant 0.45% (bootstrap standard error of 2.17%) for the 12-month holding period. The corresponding dollar excess returns for the post-Bretton Woods period are -0.11% for the 3-month holding period and 0.26% for the 12-month holding period, neither of which is statistically significant, as the bootstrap standard error is 3.21% and 1.61%, respectively.

A.2.3 Developed and Emerging Countries

Finally, we consider the sample of developed and emerging countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan, Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom), and sort currencies into five portfolios.

In particular, at the one-month horizon the average currency log excess return of the carry trade is 2.40% per year over the long sample period (1/1951 – 12/2015), which is more than offset by the local currency bond log excess return of -7.05% per year. As a result, the carry trade implemented using 10-year bonds yields a statistically significant average annualized return of 4.65% (the bootstrap standard error is 2.01%). The average inflation rate of Portfolio 1 is 4.59% and its average credit rating is 5.51 (4.96 when adjusted for outlook), whereas the average inflation rate of Portfolio 5 is 5.66% and its average credit rating is 4.70 (4.89 when adjusted for outlook). When we consider the post-Bretton Woods period (1/1975 – 12/2015), we get very similar results: the average currency log excess return is 3.04% per year, which is offset by a local currency bond log excess return of -6.36% , so the 10-year bond carry trade strategy yields a statistically significant annualized dollar return of -3.33% (bootstrap standard error of 1.29%). The average inflation rate of Portfolio 1 is 4.47% and its average credit rating is 5.45 (5.06 adjusted for outlook), whereas the average inflation rate of Portfolio 5 is 6.43% and its average credit rating is 4.78 (4.84 adjusted for outlook).

When we increase the holding period to 3 or 12 months, similar results emerge. For the long sample (1/1951 – 12/2015), the annualized dollar excess return of the carry trade strategy implemented using 10-year bonds is a non-significant -2.11% (bootstrap standard error of 2.07%) for the 3-month horizon and a non-significant -0.63% (bootstrap standard error of 2.18%) for the 12-month horizon. The corresponding dollar excess returns for the post-Bretton Woods period are -1.63% for the 3-month holding period and -0.70% for the 12-month holding period, both of which are marginally significant (bootstrap standard error of 1.47% and 1.62%, respectively).

A.3 Portfolio Cross-Sectional Evidence: Sorting by Interest Rate Levels

We now turn to currency portfolios sorted on interest rate levels.

A.3.1 Benchmark Sample

Figure 7 plots the composition of the three interest rate-sorted portfolios of the currencies of the benchmark sample, ranked from low (Portfolio 1) to high (Portfolio 3) interest rate currencies. Typically, Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are the carry trade investment currencies in Portfolio 3. The other currencies switch between portfolios quite often.

Over the entire 1/1951 – 12/2015 period, the average currency log excess return of the carry trade is 3.23% per year, whereas the local currency bond log excess return is -2.55% per year. As a result, the interest rate carry trade implemented using 10-year bonds yields an average annualized return of 0.68%, which is not statistically significant, as its bootstrap standard error is 1.07%. The average inflation rate of Portfolio 1 is 2.81% and its average credit rating is 1.33 (1.39 when adjusted for outlook), whereas the average inflation rate of Portfolio 3 is 5.15% and its average credit rating is 1.57 (1.92 when adjusted for outlook). Our findings are very similar when we consider only the post-Bretton Woods period (1/1975 – 12/2015): the average currency log excess return is 3.50% per year, largely offset by a local currency bond log excess return of -2.51% , so the 10-year bond carry trade strategy yields a statistically not significant annualized dollar return of 0.99% (bootstrap standard error of 1.57%). The average inflation rate of Portfolio 1 is 2.00% and its average credit rating is 1.36 (1.41 when adjusted for outlook), whereas the average inflation rate of Portfolio 3 is 5.32% and its average credit rating is 1.60 (1.93 when adjusted for outlook).

We find very similar results when we increase the holding period: there is no evidence of statistically significant differences in dollar bond risk premia across the currency portfolios. In particular, for the entire 1/1951 – 12/2015 period, the annualized dollar excess return of the carry trade strategy implemented using 10-year bonds is a non-significant 1.03% (bootstrap standard error of 1.12%) for the 3-month holding period and a non-significant 1.23% (bootstrap standard error of 1.20%) for the 12-month holding period. The corresponding dollar excess returns for the post-Bretton Woods period are 1.15% for the 3-month holding period and

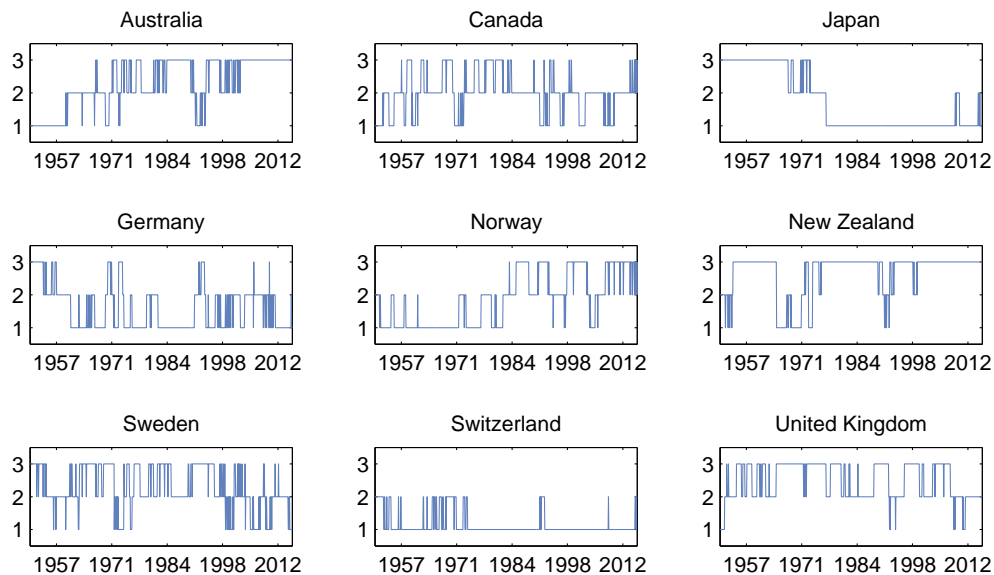


Figure 7: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 9 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 1/1951 to 12/2015.

1.18% for the 12-month holding period, neither of which is statistically significant (bootstrap standard error of 1.65% and 1.69%, respectively).

A.3.2 Developed Countries

We now turn to the sample of 20 developed countries. Figure 8 plots the composition of the four interest rate-sorted currency portfolios. As we can see, Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are carry trade investment currencies in Portfolio 4.

We start with 1-month holding period returns. Over the long sample period (1/1951 – 12/2015), the average currency log excess return of the carry trade is 2.73% per year, whereas the local currency bond log excess return is -2.15% per year. Therefore, the interest rate carry trade implemented using 10-year bonds yields a non-statistically significant average dollar annualized return of 0.58% (the bootstrap standard error is 0.90%). The average inflation rate of Portfolio 1 is 3.04% and its average credit rating is 1.50 (1.54 when adjusted for outlook); the average inflation rate of Portfolio 4 is 5.73% and its average credit rating is 2.93 (3.02 when adjusted for outlook). We get very similar results when we focus on the post-Bretton Woods sample: the average currency log excess return is 2.81% per year, offset by a local currency bond log excess return of -1.37% , so the 10-year bond carry trade strategy yields a statistically not significant annualized return of 1.44% (bootstrap standard error of 1.33%). The average inflation rate of Portfolio 1 is 2.30% and its average credit rating is 1.55 (1.61 adjusted for outlook), whereas the average inflation rate of Portfolio 4 is 6.07% and its average credit rating is 2.97 (3.03 adjusted for outlook).

When we increase the holding period, we get very similar results. For the 1/1951 – 12/2015 sample, the annualized dollar excess return of the carry trade strategy implemented using 10-year bonds is a non-significant 1.15% (bootstrap standard error of 0.94%) for the 3-month holding period and a non-significant 1.48% (bootstrap standard error of 0.99%) for the 12-month holding period. The corresponding dollar excess returns for the post-Bretton Woods period are 1.92% for the 3-month holding period and 1.90% for the 12-month holding period, neither of which is statistically significant, as the bootstrap standard errors are 1.37% and 1.50%, respectively.

A.3.3 Developed and Emerging Countries

Finally, we consider the sample of developed and emerging countries and sort currencies into five portfolios.

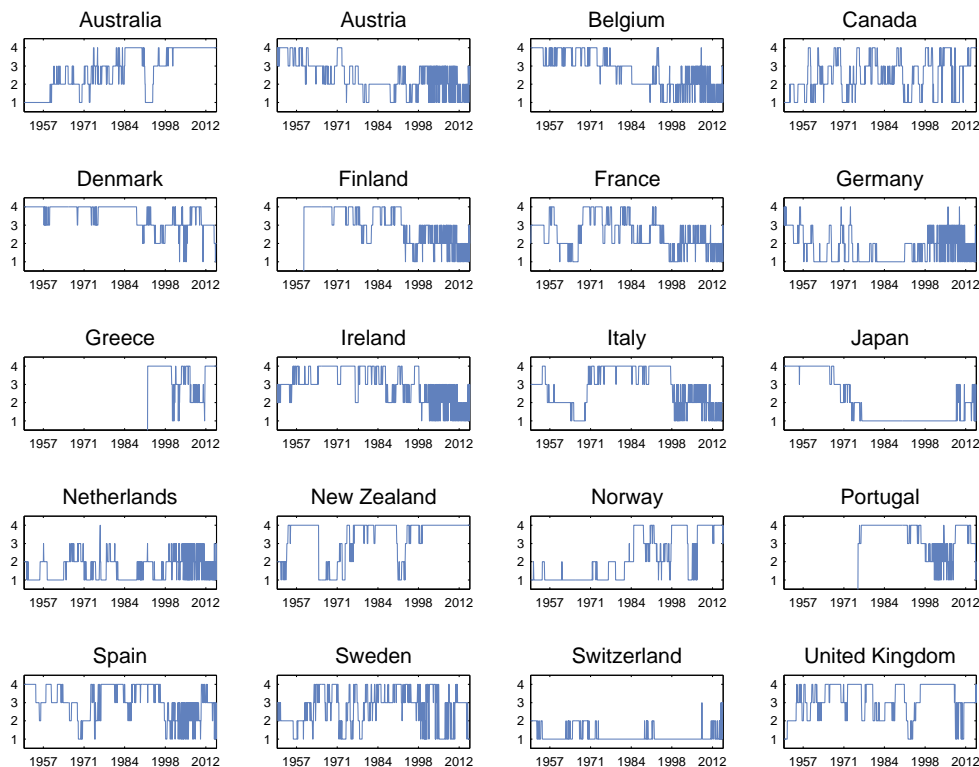


Figure 8: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 20 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 1/1951 to 12/2015.

We start by focusing on one-month returns. Over the long sample period (1/1951 – 12/2015), the average currency log excess return of the carry trade is 4.92% per year, largely offset by the local currency bond log excess return of -4.18% per year. As a result, the interest rate carry trade implemented using 10-year bonds yields a non-statistically significant average annualized return of 0.74% (the bootstrap standard error is 0.90%). The average inflation rate of Portfolio 1 is 3.17% and its average credit rating is 2.91 (2.75 when adjusted for outlook), whereas the average inflation rate of Portfolio 5 is 6.82% and its average credit rating is 6.59 (6.07 when adjusted for outlook). When we focus on the post-Bretton Woods sample, our findings are very similar: the average currency log excess return is 5.73% per year, which is offset by a local currency bond log excess return of -3.80% , so the 10-year bond carry trade strategy yields a statistically non-significant annualized return of 1.92% (the bootstrap standard error is 1.33%). The average inflation rate of Portfolio 1 is 2.49% and its average credit rating is 2.95 (2.90 adjusted for outlook); the average inflation rate of Portfolio 5 is 7.78% and its average credit rating is 6.60 (6.03 adjusted for outlook).

We now consider longer holding periods. For the long sample (1/1951 – 12/2015), the annualized dollar excess return of the carry trade strategy implemented using 10-year bonds is a non-significant 1.33% (bootstrap standard error of 1.01%) for the 3-month horizon and a marginally significant 1.94% (bootstrap standard error of 1.10%) for the 12-month horizon. The corresponding dollar excess returns for the post-Bretton Woods period are 2.56% for the 3-month holding period and 2.80% for the 12-month holding period, both of which are marginally significant (bootstrap standard error of 1.50% and 1.69%, respectively).

A.4 Portfolio Cross-Sectional Evidence: Sorting by Yield Curve Slopes

This section presents additional evidence on slope-sorted portfolios, again considering first our benchmark sample of G10 countries before turning to larger sets of developed and emerging countries.

A.4.1 Benchmark Sample

Figure 9 presents the composition over time of portfolios of the 9 currencies of the benchmark sample sorted by the slope of the yield curve.

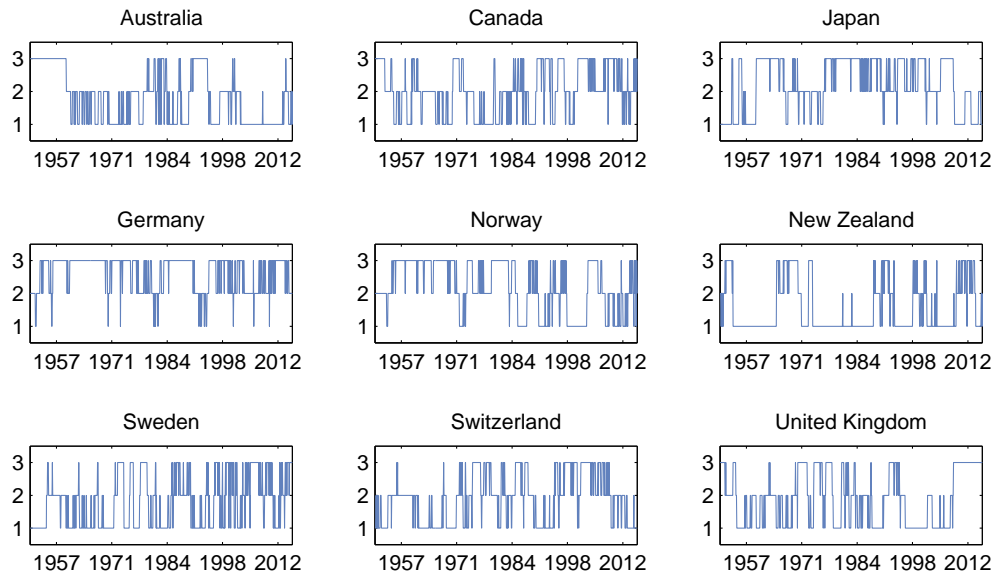


Figure 9: Composition of Slope-Sorted Portfolios — The figure presents the composition of portfolios of the currencies in the benchmark sample sorted by the slope of their yield curves. The portfolios are rebalanced monthly. The slope of the yield curve is measured by the spread between the 10-year bond yield and the one-month interest rate. Data are monthly, from 1/1951 to 12/2015.

Figure 10 presents the cumulative one-month log excess returns on investments in foreign Treasury bills and foreign 10-year bonds, starting in 1951. The returns correspond to an investment strategy going long in Portfolio 1 (flat yield curves, mostly high short-term interest rates) and short in Portfolio 3 (steep yield curves, mostly low short-term interest rates). Over the entire 1/1951 – 12/2015 period, the average currency log excess return of the slope carry trade is 3.01% per year, whereas the local currency bond log excess return is -5.46% per year. Therefore, the slope carry trade implemented using 10-year bonds results in an average return of -2.45% per year, which is statistically significant (bootstrap standard error of 0.98%). It is worth noting that neither inflation risk nor credit risk seem to be able to explain this offsetting effect: the average inflation rate of Portfolio 1, which has a low average term premium, is 4.71% and its average credit rating is 1.52 (1.84 when adjusted for outlook), whereas the average inflation rate of Portfolio 3, which has a high average term premium, is 3.51% and its average credit rating is 1.28 (1.37 when adjusted for outlook). As seen in Table 2, we get similar results when we focus only on the post-Bretton Woods period (1/1975 – 12/2015). Finally, we consider the 7/1989 – 12/2015 sample period. The one-month average currency excess return of the slope carry trade strategy is 4.41%, largely offset by the local currency bond excess return of -3.40% . As a result, the average dollar bond excess return is 1.02%, which is not statistically significant, as its bootstrap standard error is 1.32%. Portfolio 1 has an average inflation rate of 2.31% and an average credit rating of 1.71 (1.75 when adjusted for outlook), whereas Portfolio 3 has an average inflation rate of 1.51% and an average credit rating of 1.43 (1.49 when adjusted for outlook).

We now consider longer holding periods. Overall, we find no evidence of statistically significant differences in dollar bond risk premia across the currency portfolios. For the full 1/1951 – 12/2015 period, the annualized dollar excess return of the slope carry trade strategy implemented using 10-year bonds is a non-significant -1.58% (bootstrap standard error of 0.99%) for the 3-month holding period, as the average currency risk premium of 2.53% is more than offset by the average local currency term premium of -4.12% . For the 12-month holding period, the average currency risk premium is 1.98%, which is offset by the average local currency term premium of -3.15% , yielding an average non-significant dollar term premium of -1.17% (bootstrap standard error of 1.00%). The corresponding dollar excess returns for the post-Bretton Woods period (1/1975 – 12/2015) are -0.88% for the 3-month holding period (average currency risk premium of 2.95%, average local currency term premium of -3.83%) and -0.50% for the 12-month holding period (average currency risk premium of 2.19%, average local currency term premium of -2.68%), neither which are is

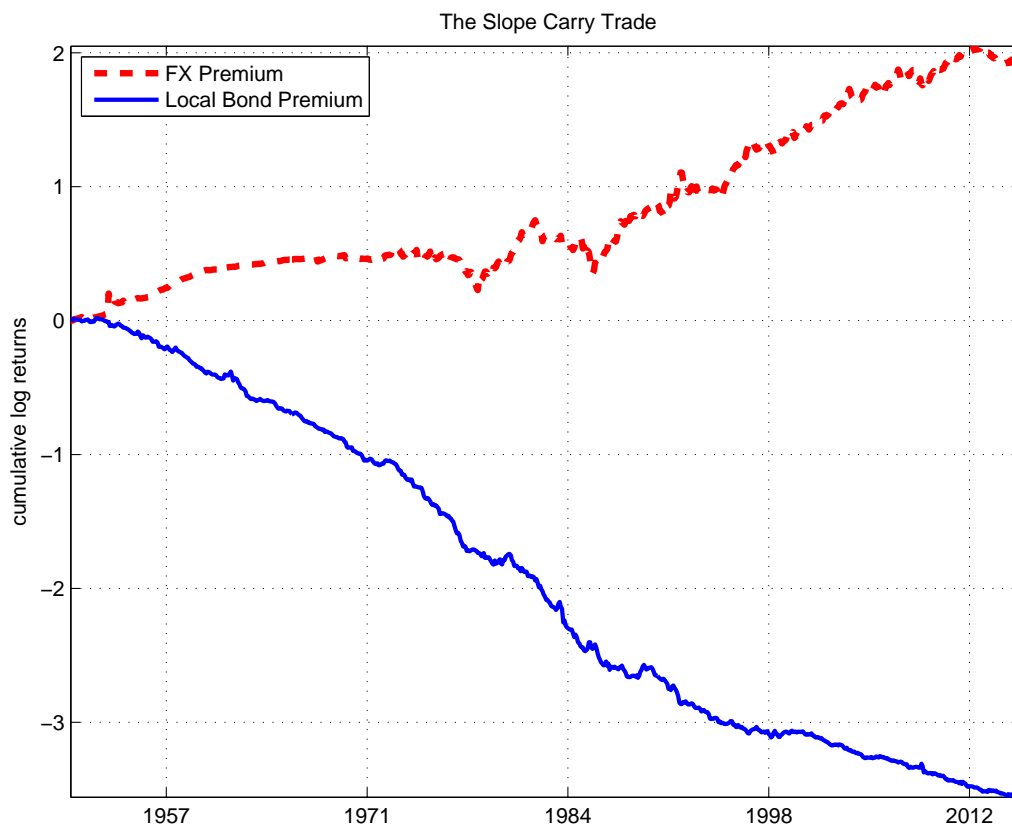


Figure 10: The Carry Trade and Term Premia: Conditional on the Slope of the Yield Curve – The figure presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. The benchmark panel of countries includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. Countries are sorted every month by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the spread between the 10-year bond yield and the one-month interest rate. The returns correspond to an investment strategy going long in Portfolio 1 and short in the Portfolio 3. The sample period is 1/1951–12/2015.

significant, as the bootstrap standard error is 1.43% and 1.46%, respectively. Finally, we turn to the 7/1989 – 12/2015 period. The average dollar bond premium is 0.98% for the 3-month horizon (average currency risk premium of 3.14%, average local currency bond premium of -2.16%) and 1.35% for the 12-month horizon (average currency risk premium of 2.75%, average local currency bond premium of -1.39%). Both of those dollar bond premia are non-significant, as their bootstrap standard error is 1.52% and 1.71%, respectively.

A.4.2 Developed Countries

In the sample of developed countries, the flat-slope currencies (Portfolio 1) are typically those of Australia, New Zealand, Denmark and the U.K., while the steep-slope currencies (Portfolio 4) are typically those of Germany, the Netherlands, and Japan. The portfolio compositions are plotted in Figure 11.

At the one-month horizon, the 2.50% spread in currency excess returns obtained in the full sample period (1/1951 – 12/2015) is more than offset by the -6.73% spread in local term premia. This produces a statistically significant average dollar excess return of -4.22% (bootstrap standard error of 1.02%) on a position that is long in the high yielding, low slope currencies (Portfolio 1) and short in the low yielding, high slope currencies (Portfolio 4). The average inflation rate of Portfolio 1 is 5.13% and its average credit rating is 2.20 (2.34 when adjusted for outlook), whereas the average inflation rate of Portfolio 4 is 3.97% and its average

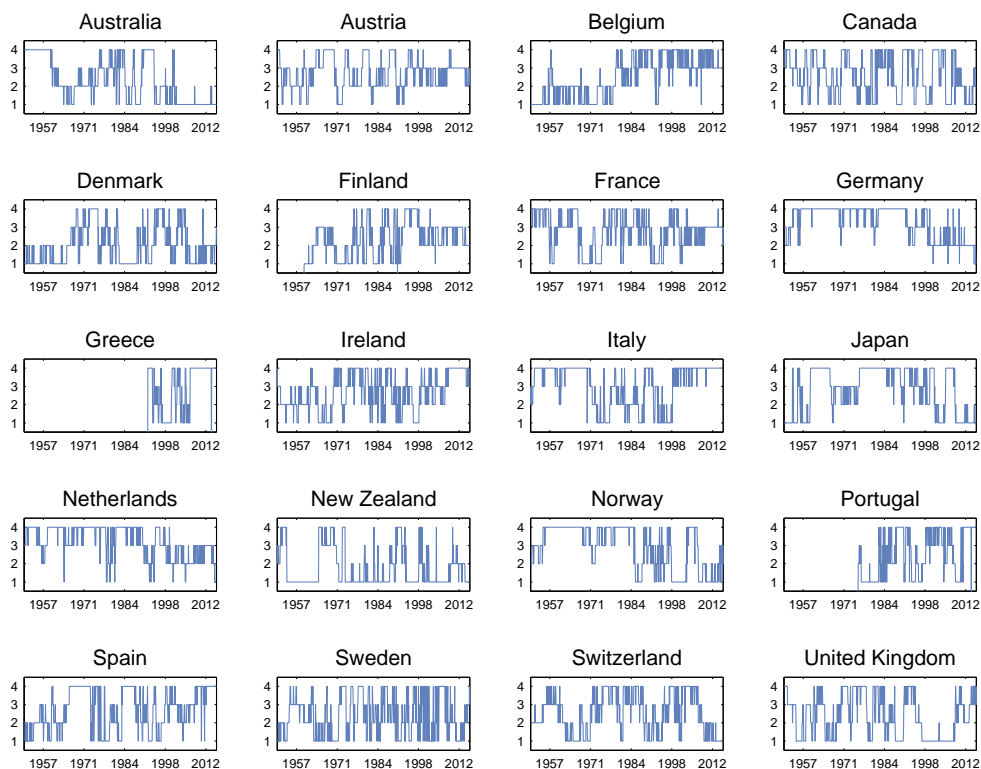


Figure 11: Composition of Slope-Sorted Portfolios — The figure presents the composition of portfolios of 20 currencies sorted by their yield curve slopes. The portfolios are rebalanced monthly. Data are monthly, from 1/1951 to 12/2015.

credit rating is 2.88 (2.97 when adjusted for outlook). Those results are essentially unchanged in the post-Bretton Woods period: the average currency excess return is 3.04%, more than offset by the average local currency bond excess return of -7.60% , so the slope carry trade yields an average excess return of -4.56% , which is statistically significant (bootstrap standard error of 1.48%). The average inflation rate of Portfolio 1 is 5.36% and its average credit rating is 2.21 (2.34 when adjusted for outlook), whereas the average inflation rate of Portfolio 4 is 3.49% and its average credit rating is 3.04 (3.16 when adjusted for outlook).

We now turn to longer holding periods. In the 3-month horizon, investing in Portfolio 1 and shorting Portfolio 4 during the long sample period (1/1951 – 12/2015) yields an average currency excess return of 2.03% and an average local currency bond excess return of -5.13% , resulting in a statistically significant dollar bond excess return of -3.10% (bootstrap standard error of 1.11%). In the same period, the 12-month average currency excess return is 1.86% and the average local currency bond excess return is -3.53% , so the average dollar bond excess return is a non-significant -1.67% (bootstrap standard error of 1.42%). Similar results emerge when we focus on the post-Bretton Woods period. In the 3-month horizon, the average currency excess return is 2.31% and the average local currency bond excess return is -5.32% , yielding an average dollar bond excess return of -3.00% , which is marginally statistically significant (bootstrap standard error of 1.63%). In the 12-month horizon, the average currency excess return is 1.90% and the average local currency bond excess return is -3.42% , so the average dollar bond excess return is a non-significant -1.52% (bootstrap standard error of 2.22%).

A.4.3 Developed and Emerging Countries

In the entire sample of countries, the difference in currency risk premia at the one-month horizon is 3.44% per year, which is more than offset by a -9.84% difference in local currency term premia. As a result, investors earn a statistically significant -6.41% per annum (the bootstrap standard error is 1.06%) on a long-short bond position. As before, this involves going long the bonds of flat-yield-curve currencies (Portfolio 1), typically high interest rate currencies, and shorting the bonds of the steep-slope currencies

(Portfolio 5), typically the low interest rate ones. The average inflation rate of Portfolio 1 is 5.77% and its average credit rating is 4.77 (4.74 when adjusted for outlook), whereas the average inflation rate of Portfolio 5 is 4.54% and its average credit rating is 5.62 (5.33 when adjusted for outlook). When we focus on the post-Bretton Woods period (1/1975 – 12/2015), we get very similar results: the average currency log excess return is 4.59% per year, which is more than offset by a local currency bond log excess return difference of -11.53% , so the 10-year bond carry trade strategy yields a statistically significant annualized return of -6.94% (bootstrap standard error of 1.51%). The average inflation rate of Portfolio 1 is 6.16% and its average credit rating is 4.79 (4.69 adjusted for outlook), whereas the average inflation rate of Portfolio 5 is 4.43% and its average credit rating is 5.73 (5.55 adjusted for outlook).

When we increase the holding period to 3 or 12 months, similar results emerge. For the long sample (1/1951 – 12/2015), the average annualized dollar excess return of the slope carry trade strategy (long Portfolio 1, short Portfolio 5) implemented using 10-year bonds is a statistically significant -5.32% (bootstrap standard error of 1.17%) for the 3-month horizon: the average currency excess return is 2.76%, more than offset by the average local currency bond excess return of -8.08% . For the 12-month horizon, the average currency excess return is 2.47% and the local currency bond excess return is -5.48% , so the average dollar excess return for the slope carry trade is -3.01% (statistically significant, as the bootstrap standard error is 1.29%). Finally, for the post-Bretton Woods period, the average 3-month currency excess return is 3.55% and the average local currency bond excess return is -9.22% , so the dollar excess return of the slope carry trade is -5.66% (statistically significant, as the bootstrap standard error is 1.73%). For the same period, the average 12-month currency excess return is 3.06% and the average local currency bond excess return is -5.83% , resulting in an average dollar excess return of -2.78% (not significant, given a bootstrap standard error of 1.97%).

A.5 Foreign Bond Returns Across Maturities

This section reports additional results obtained with zero-coupon bonds. We start with the bond risk premia in our benchmark sample of G10 countries and then turn to a larger set of developed countries. We then show that holding period returns on zero-coupon bonds, once converted to a common currency (the U.S. dollar, in particular), become increasingly similar as bond maturities approach infinity.

A.5.1 Benchmark Countries

Figure 12 reports results for all maturities. The figure shows the local currency bond log excess returns in the top panels, the currency log excess returns in the middle panels, and the dollar bond log excess returns in the bottom panels. The top panels show that countries with the steepest local yield curves (Portfolio 3, center) exhibit local bond excess returns that are higher, and increase faster with maturity, than the flat yield curve countries (Portfolio 1, left). Thus, ignoring the effect of exchange rates, investors should invest in the short-term and long-term bonds of steep yield curve currencies.

Including the effect of currency fluctuations, by focusing on dollar returns, radically alters the results. The bottom panels of Figure 12 show that the dollar excess returns of Portfolio 1 are on average higher than those of Portfolio 3 at the short end of the yield curve, consistent with the carry trade results of Ang and Chen (2010). Yet, an investor who would attempt to replicate the short-maturity carry trade strategy at the long end of the maturity curve would incur losses on average: the long-maturity excess returns of flat yield curve currencies are lower than those of steep yield curve currencies, as currency risk premia more than offset term premia. This result is apparent in the lower panel on the right, which is the same as Figure 1 in the main text.

A.5.2 Developed Countries

When we tuning to the entire sample of developed countries, the results are very similar to those attained in our benchmark sample. An investor who buys the short-term bonds of flat-yield curve currencies and shorts the one-year bonds of steep-yield-curve currencies realizes a statistically significant dollar excess return of 3.41% per year on average (bootstrap standard error of 1.41%). However, at the long end of the maturity structure, this strategy generates negative and insignificant excess returns: the average annualized dollar excess return of an investor who pursues this strategy using 15-year bonds is -2.30% (bootstrap standard error of 2.48%). Our findings are presented graphically in Figure 13, which shows the local currency bond log excess returns in the top panels, the currency log excess returns in the middle panels, and the dollar bond log excess returns in the bottom panels as a function of maturity.

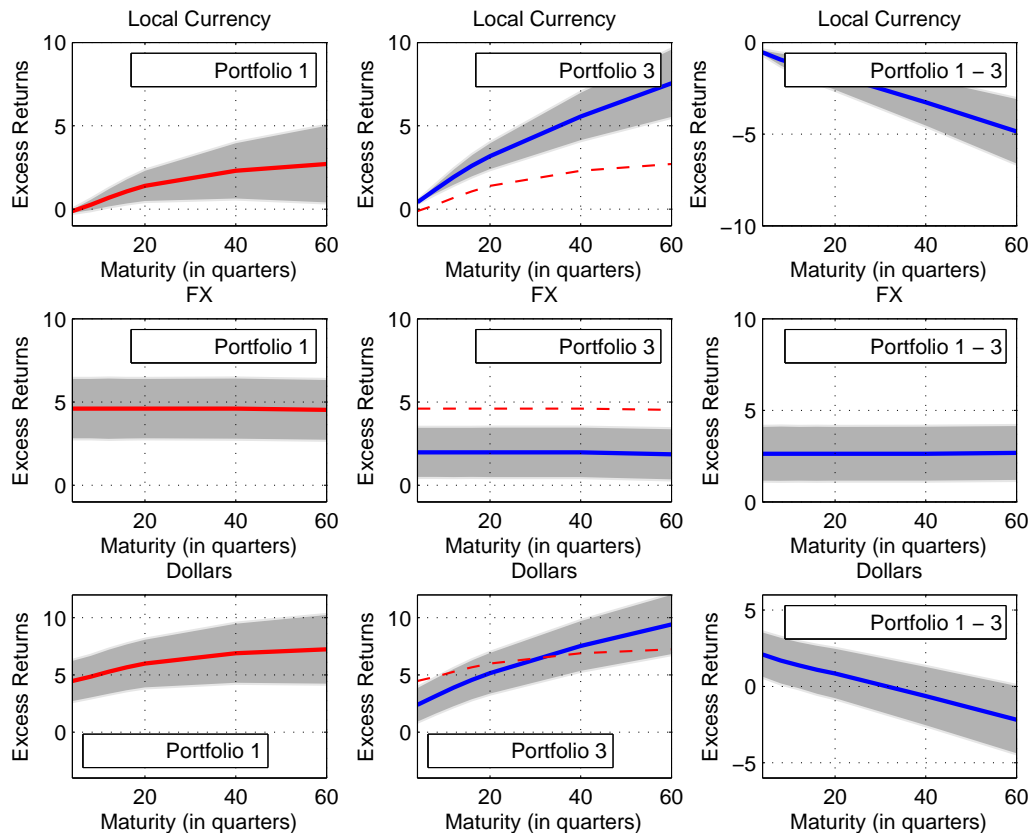


Figure 12: Dollar Bond Risk Premia Across Maturities— The figure shows the log excess returns on foreign bonds in local currency in the top panel, the currency excess return in the middle panel, and the log excess returns on foreign bonds in U.S. dollars in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 3 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 4/1985–12/2015. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t . The holding period is one quarter. The returns are annualized. The shaded areas correspond to one standard deviation above and below each point estimate. Standard deviations are obtained by bootstrapping 10,000 samples of non-overlapping returns.

B Finite vs. Infinite Maturity Bond Returns

Our empirical results pertain to 10- and 15-year bond returns while our theoretical results pertain to infinite-maturity bonds. This discrepancy raises the question of the theoretical validity of our empirical analysis. To address this question, we use the state-of-the-art Joslin, Singleton, and Zhu (2011) term structure model to study empirically the difference between the 10-year and infinite-maturity bonds. In particular, we estimate a version of the Joslin, Singleton, and Zhu (2011) term structure model with three factors, the three first principal components of the yield covariance matrix.²¹ This Gaussian dynamic term structure model is estimated on zero-coupon rates over the period from April 1985 to December 2015, the same period used in our empirical work, for each country in our benchmark sample. Each country-specific model is estimated independently, without using any exchange rate data. The maturities considered are 6 months, and 1, 2, 3, 5, 7, and 10 years. Using the parameter estimates, we derive the implied bond returns for different maturities. We report simulated data for Australia, Canada, Germany, Japan, Norway, Switzerland, U.K., and U.S. and ignore the simulated data for New Zealand and Sweden as the parameter estimates imply that

²¹We thank the authors for making their code available on their web pages.

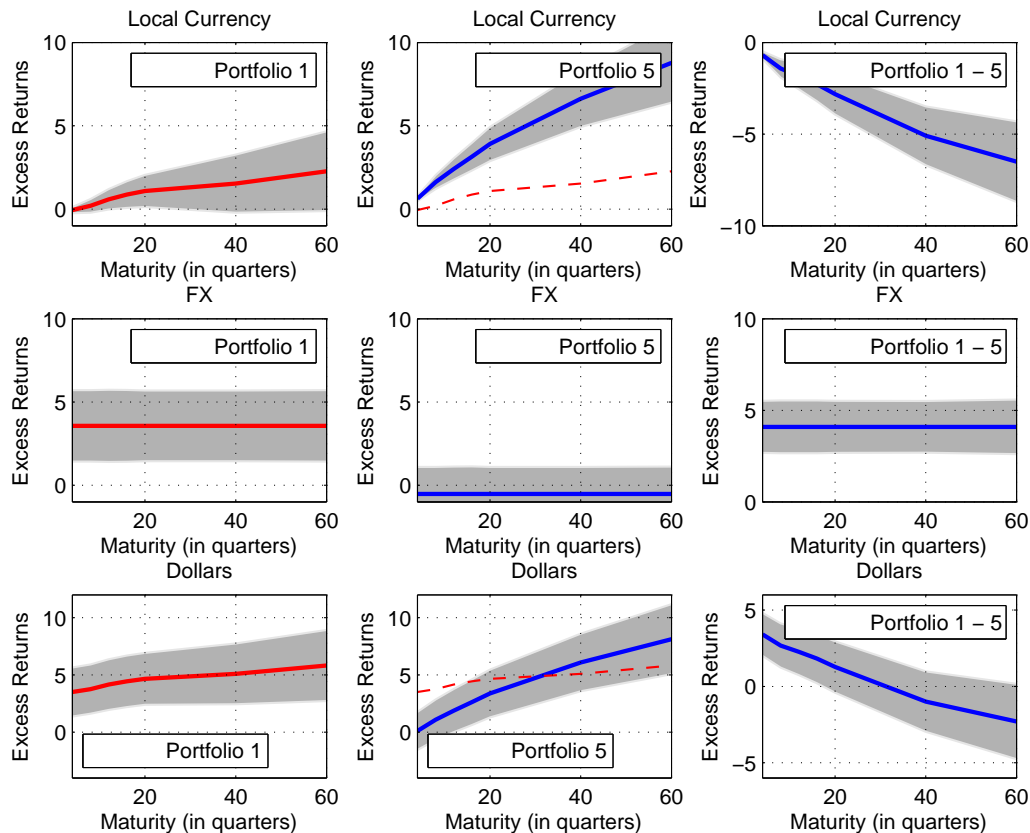


Figure 13: Dollar Bond Risk Premia Across Maturities:-: Extended Sample — The figure shows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 5 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 5/1987–12/2015. The unbalanced sample includes Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Hungary, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into five portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t . The holding period is one quarter. The returns are annualized. The shaded areas correspond to one standard deviation above and below each point estimate. Standard deviations are obtained by bootstrapping 10,000 samples of non-overlapping returns.

bond yields turn sharply negative on long maturities for those two countries. We study both unconditional and conditional returns, forming portfolios of countries sorted by the level or slope of their yield curves, as we did in the data. Table 8 reports the simulated moments.

We first consider the unconditional holding period bond returns across countries. The average (annualized) log return on the 10-year bond is lower than the log return on the infinite-maturity bond for all countries except Australia, the U.K., and the U.S., but the differences are not statistically significant, except for Japan. The unconditional correlation between the two log returns ranges from 0.88 to 0.96 across countries; for example, it is 0.89 for the U.S. Furthermore, the estimations imply very volatile log SDFs that exhibit little correlation across countries. As a result, the implied exchange rate changes are much more volatile than in the data. We then turn to conditional bond returns, obtained by sorting countries into two portfolios, either by the level of their short-term interest rate or by the slope of their yield curve. The portfolio sorts recover the results highlighted in the previous section: low (high) short-term interest rates correspond to high (low) average local bond returns. Likewise, low (high) slopes correspond to low (high) average local bond returns. The infinite maturity bonds tend to offer larger conditional returns than the 10-year bonds, but the differences are not significant. The correlation between the conditional returns of the 10-year and infinite maturity bond

portfolios ranges from 0.86 to 0.93 across portfolios.

In theory, it is certainly possible to write a model where the 10-year bond returns, once expressed in the same currency, offer similar average returns across countries (as we find in the data), while the infinite maturity bonds do not. In that case, there would be a gap between our theory and the data. In such a model, however, exchange rates would have unit root components driven by common shocks and the cross-sectional distribution of exchange rates would fan out over time. For developing countries with strong trade links and similar inflation rates, this seems hard to defend. Moreover, although we cannot rule out its existence, we do not know of such a model. In the state-of-the-art of the term structure modeling, our inference about infinite-maturity bonds from 10-year bonds is reasonable.

Table 8: Simulated Bond Returns

Panel A: Country Returns								
	US	Australia	Canada	Germany	Japan	Norway	Switzerland	UK
$y^{(10)}$ (data)	5.58	6.97	5.81	4.97	2.77	4.26	3.17	6.10
$y^{(10)}$	5.58	6.97	5.81	4.97	2.77	4.26	3.18	6.09
$rx^{(10)}$	5.60	4.50	4.53	4.33	4.05	3.14	2.95	3.50
s.e.	[1.43]	[1.71]	[1.45]	[1.17]	[1.13]	[1.71]	[1.12]	[1.52]
$rx^{(\infty)}$	-0.44	2.17	6.69	6.33	7.38	5.96	6.42	2.74
s.e.	[10.87]	[10.38]	[8.47]	[2.33]	[2.46]	[3.89]	[3.23]	[4.52]
Corr ($rx^{(10)}, rx^{(\infty)}$)	0.89	0.92	0.89	0.92	0.93	0.96	0.93	0.88
$rx^{(\infty)} - rx^{(10)}$	-6.04	-2.33	2.16	2.00	3.33	2.83	3.47	-0.76
s.e.	[9.63]	[8.77]	[6.98]	[1.34]	[1.46]	[2.30]	[2.22]	[3.33]
σ_{m^*}	239.17	241.92	127.14	118.45	211.76	132.76	227.59	153.22
$corr(m, m^*)$	1.00	0.01	0.33	0.20	0.03	0.05	0.14	0.03
$\sigma_{\Delta s}$		310.81	202.65	244.63	314.14	190.44	271.17	279.99
Panel B: Portfolio Returns								
	Sorted by Level		Sorted by Slope					
Sorting variable (level/slope)		2.57	5.60		0.15	1.81		
$rx^{(10)}$		4.10	4.52		3.00	5.48		
s.e.		[1.05]	[1.24]		[1.13]	[1.23]		
$rx^{(\infty)}$		4.48	6.00		0.79	9.61		
s.e.		[4.17]	[5.62]		[4.33]	[5.90]		
Corr ($rx^{(10)}, rx^{(\infty)}$)		0.86	0.93		0.89	0.90		
$rx^{(\infty)} - rx^{(10)}$		0.38	1.48		-2.21	4.13		
s.e.		[3.28]	[4.58]		[3.40]	[4.84]		

Notes: Panel A reports moments on simulated data at the country level. For each country, the table first compares the 10-year yield in the data and in the model, and then reports the annualized average simulated log excess return (in percentage terms) of bonds with maturities of 10 years and infinity, as well as the correlation between the two bond returns. The table also reports the annualized volatility of the log SDF, the correlation between the foreign log SDF and the U.S. log SDF, and the annualized volatility of the implied exchange rate changes. Panel B reports conditional moments obtained by sorting countries by either the level of their short-term interest rates or the slope of their yield curves into two portfolios. The table reports the average value of the sorting variable, and then the average returns on the 10-year and infinite-maturity bonds, along with their correlation. The simulated data come from the benchmark 3-factor model (denoted RPC) in Joslin, Singleton, and Zhu (2011) that sets the first 3 principal components of bond yields as the pricing factors. The model is estimated on zero-coupon rates for Germany, Japan, Norway, Switzerland, U.K., and U.S. The sample estimation period is 4/1985–12/2015. The standard errors (denoted s.e. and reported between brackets) were generated by block-bootstrapping 10,000 samples of 369 monthly observations.

C Dynamic Term Structure Models

This section reports additional results on dynamic term structure models, starting with the simple Vasicek one-factor model, before turning to essentially affine k -factor models and the model studied in Lustig, Roussanov, and Verdelhan (2011).

For the reader's convenience, we repeat the three main equations that will be key to analyze the currency and bond risk premia:

$$E_t \left[rx_{t+1}^{FX} \right] = (f_t - s_t) - E_t(\Delta s_{t+1}) = L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right), \quad (21)$$

$$E_t \left[rx_{t+1}^{(\infty),*} \right] = \lim_{k \rightarrow \infty} E_t \left[rx_{t+1}^{(k),*} \right] = L_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right) - L_t \left(\frac{\Lambda_{t+1}^{*,\mathbb{P}}}{\Lambda_t^{*,\mathbb{P}}} \right), \quad (22)$$

$$E_t \left[rx_{t+1}^{(\infty),*} \right] + E_t \left[rx_{t+1}^{FX} \right] = E_t \left[rx_{t+1}^{(\infty)} \right] + L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) - L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \right). \quad (23)$$

As already noted, Equation (21) shows that the currency risk premium is equal to the difference between the entropy of the domestic and foreign SDFs (Backus, Foresi, and Telmer, 2001). Equation (22) shows that the term premium is equal to the difference between the total entropy of the SDF and the entropy of its permanent component (Alvarez and Jermann, 2005). Equation (23) shows that the foreign term premium in dollars is equal to the domestic term premium plus the difference in the entropy of the foreign and domestic permanent component of the SDFs.

C.1 Vasicek (1977)

Model In the Vasicek model, the log SDF evolves as:

$$-m_{t+1} = y_{1,t} + \frac{1}{2} \lambda^2 \sigma^2 + \lambda \varepsilon_{t+1},$$

where $y_{1,t}$ denotes the short-term interest rate. It is affine in a single factor:

$$\begin{aligned} x_{t+1} &= \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2) \\ y_{1,t} &= \delta + x_t. \end{aligned}$$

In this model, x_t is the level factor and ε_{t+1} are shocks to the level of the term structure. The Jensen term is there to ensure that $E_t(M_{t+1}) = \exp(-y_{1,t})$. Bond prices are exponentially affine. For any maturity n , bond prices are equal to $P_t^{(n)} = \exp(-B_0^n - B_1^n x_t)$. The price of the one-period risk-free note ($n = 1$) is naturally:

$$P_t^{(1)} = \exp(-y_{1,t}) = \exp(-B_0^1 - B_1^1 x_t),$$

with $B_0^1 = \delta$, $B_1^1 = 1$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t \left(M_{t+1} P_{t+1}^{(n-1)} \right)$, which implies:

$$-B_0^n - B_1^n x_t = -\delta - x_t - B_0^{n-1} - B_1^{n-1} \rho x_t + \frac{1}{2} (B_1^{n-1})^2 \sigma^2 x_t + \lambda B_1^{n-1} \sigma^2.$$

The coefficients B_0^n and B_1^n satisfy the following recursions:

$$\begin{aligned} B_0^n &= \delta + B_0^{n-1} - \frac{1}{2} \sigma^2 (B_1^{n-1})^2 - \lambda B_1^{n-1} \sigma^2, \\ B_1^n &= 1 + B_1^{n-1} \rho. \end{aligned}$$

Decomposition (Alvarez and Jermann, 2005) We first implement the Alvarez and Jermann (2005) approach. The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n \rightarrow \infty} \frac{\beta^{t+n}}{P_t^n} = \lim_{n \rightarrow \infty} \beta^{t+n} e^{B_0^n + B_1^n x_t},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$0 < \lim_{n \rightarrow \infty} \frac{P_t^n}{\beta^n} < \infty.$$

The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n \rightarrow \infty} B_0^n - B_0^{n-1} = \delta - \frac{1}{2}\sigma^2(B_1^\infty)^2 - \lambda B_1^\infty \sigma^2$, where B_1^∞ is $1/(1-\rho)$. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\delta + \frac{1}{2}\sigma^2(B_1^\infty)^2 + \lambda B_1^\infty \sigma^2}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the pricing kernel is thus equal to:

$$\frac{\Lambda_{t+1}^T}{\Lambda_t^T} = \beta e^{B_1^\infty(x_{t+1}-x_t)} = \beta e^{\frac{1}{1-\rho}(\rho-1)x_t + \frac{1}{1-\rho}\varepsilon_{t+1}} = \beta e^{-x_t + \frac{1}{1-\rho}\varepsilon_{t+1}}.$$

The martingale component of the pricing kernel is then:

$$\frac{\Lambda_{t+1}^P}{\Lambda_t^P} = \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\Lambda_{t+1}^T}{\Lambda_t^T} \right)^{-1} = \beta^{-1} e^{x_t - \frac{1}{1-\rho}\varepsilon_{t+1} - \delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1}} = \beta^{-1} e^{-\delta - \frac{1}{2}\lambda^2\sigma^2 - (\frac{1}{1-\rho} + \lambda)\varepsilon_{t+1}}.$$

In the case of $\lambda = -B_1^\infty = -\frac{1}{1-\rho}$, the martingale component of the pricing kernel is constant and all the shocks that affect the pricing kernel are transitory.

Decomposition (Hansen and Scheinkman (2009)) We now show that the Hansen and Scheinkman (2009) methodology leads to similar results. Guess an eigenfunction ϕ of the form

$$\phi(x) = e^{cx}$$

where c is a constant. Then, the (one-period) eigenfunction problem can be written as

$$E_t \left[\exp(-\delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1} + cx_{t+1}) \right] = \exp(\beta + cx_t).$$

Expanding and matching coefficients, we solve for the constants c and β :

$$c = -\frac{1}{1-\rho}$$

$$\beta = -\delta + \frac{1}{2}\sigma^2 \left(\frac{1}{1-\rho} \right)^2 + \lambda\sigma^2 \left(\frac{1}{1-\rho} \right)$$

As shown above, the recursive definition of the bond price coefficients B_0^n and B_1^n implies that:

$$c = -B_1^\infty.$$

The transitory component of the pricing kernel is by definition:

$$\Lambda_t^T = e^{\beta t - cx_t}$$

The transitory and permanent SDF component are thus:

$$\frac{\Lambda_{t+1}^T}{\Lambda_t^T} = e^{\beta - c(x_{t+1} - x_t)} = e^{\beta - x_t + \frac{1}{1-\rho}\varepsilon_{t+1}}$$

$$\frac{\Lambda_{t+1}^P}{\Lambda_t^P} = \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\Lambda_{t+1}^T}{\Lambda_t^T} \right)^{-1} = e^{-\delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1}} e^{-\beta + x_t - \frac{1}{1-\rho}\varepsilon_{t+1}} = e^{-\left[\frac{1}{2} \left(\frac{1}{1-\rho} \right)^2 + \lambda \frac{1}{1-\rho} + \frac{1}{2}\lambda^2 \right] \sigma^2 - \left(\frac{1}{1-\rho} + \lambda \right) \varepsilon_{t+1}}$$

If $\lambda = -\frac{1}{1-\rho}$, then the martingale SDF component becomes

$$\frac{\Lambda_{t+1}^P}{\Lambda_t^P} = 1$$

so the entirety of the SDF is its transitory component.

Term and Risk Premium The expected log excess return of an infinite maturity bond is then:

$$E_t[rx_{t+1}^{(\infty)}] = -\frac{1}{2}\sigma^2(B_1^\infty)^2 - \lambda B_1^\infty \sigma^2.$$

The first term is a Jensen term. The risk premium is constant and positive if λ is negative. The SDF is homoskedastic. The expected log currency excess return is therefore constant:

$$E_t[-\Delta s_{t+1}] + y_t^* - y_t = \frac{1}{2} \text{Var}_t(m_{t+1}) - \frac{1}{2} \text{Var}_t(m_{t+1}^*) = \frac{1}{2} \lambda \sigma^2 - \frac{1}{2} \lambda^* \sigma^{*2}.$$

When $\lambda = -B_1^\infty = -\frac{1}{1-\rho}$, the martingale component of the pricing kernel is constant and all the shocks that affect the pricing kernel are transitory. By using the expression for the bond risk premium in Equation (22), it is straightforward to verify that the expected log excess return of an infinite maturity bond is in this case:

$$E_t[r x_{t+1}^{(\infty)}] = \frac{1}{2} \sigma^2 \lambda^2.$$

Model with Country-Specific Factor We start by examining the case in which each country has its own factor. We assume the foreign pricing kernel has the same structure, but it is driven by a different factor with different shocks:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2} \lambda^{*2} \sigma^{*2} + \lambda^* \varepsilon_{t+1}^*, \\ x_{t+1}^* &= \rho x_t^* + \varepsilon_{t+1}^*, \quad \varepsilon_{t+1}^* \sim \mathcal{N}(0, \sigma^{*2}) \\ y_{1,t} &= \delta^* + x_t^*. \end{aligned}$$

Equation (21) shows that the expected log currency excess return is constant: $E_t[r x_{t+1}^{FX}] = \frac{1}{2} \text{Var}_t(m_{t+1}) - \frac{1}{2} \text{Var}_t(m_{t+1}^*) = \frac{1}{2} \lambda^2 \sigma^2 - \frac{1}{2} \lambda^{*2} \sigma^{*2}$.

Result 2. *In a Vasicek model with country-specific factors, the long bond uncovered return parity holds only if the model parameters satisfy the following restriction: $\lambda = -\frac{1}{1-\rho}$.*

Under these conditions, there is no martingale component in the pricing kernel and the foreign term premium on the long bond expressed in home currency is simply $E_t[r x_{t+1}^{(*, \infty)}] = \frac{1}{2} \lambda^2 \sigma^2$. This expression equals the domestic term premium. The nominal exchange rate is stationary.

Symmetric Model with Global Factor Next, we examine the case in which the single state variable x_t is global. The foreign SDF is thus:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2} \lambda^{*2} \sigma^2 + \lambda^* \varepsilon_{t+1}, \\ x_{t+1} &= \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2) \\ y_{1,t} &= \delta^* + x_t. \end{aligned}$$

This case is key for our understanding of carry risk. Since carry trade returns are base-currency-invariant and obtained on portfolios of countries that average out country-specific shocks, heterogeneity in the exposure of the pricing kernel to *global* shocks is required to explain the carry trade premium (Lustig, Roussanov, and Verdelhan, 2011). Note that here $B_1^\infty = 1/(1-\rho)$ is the same for all countries, since it only depends on the persistence of the global state variable. Likewise, $\sigma = \sigma^{*2}$ in this case.

Result 3. *In a Vasicek model with a single global factor and permanent shocks, the long bond uncovered return parity condition holds only if the countries' SDFs share the same exposure (λ) to the global shocks.*

If countries SDFs share the same parameter λ , then the permanent components of their SDFs are perfectly correlated. In this case, the result is trivial, because the currency risk premium is zero, and the local term premia are identical across countries. We now turn to a model where the currency risk premium is potentially time-varying.

C.2 Cox, Ingersoll, and Ross (1985) Model

Model The Cox, Ingersoll, and Ross (1985) model (denoted CIR) is defined by the following two equations:

$$\begin{aligned} -\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma z_t} u_{t+1}, \\ z_{t+1} &= (1 - \phi)\theta + \phi z_t - \sigma \sqrt{z_t} u_{t+1}, \end{aligned} \tag{24}$$

where M denotes the stochastic discount factor. In this model, log bond prices are affine in the state variable z : $p_t^{(n)} = -B_0^n - B_1^n z_t$. The price of a one period-bond is: $P^{(1)} = E_t(M_{t+1}) = e^{-\alpha - (\chi - \frac{1}{2}\gamma)z_t}$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t(M_{t+1}P_{t+1}^{(n-1)})$. Thus the bond price coefficients evolve according to the following second-order difference equations:

$$\begin{aligned} B_0^n &= \alpha + B_0^{n-1} + B_1^{n-1}(1 - \phi)\theta, \\ B_1^n &= \chi - \frac{1}{2}\gamma + B_1^{n-1}\phi - \frac{1}{2}(B_1^{n-1})^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}. \end{aligned} \quad (25)$$

Decomposition (Alvarez and Jermann, 2005) We first implement the Alvarez and Jermann (2005) approach. The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n \rightarrow \infty} \frac{\beta^{t+n}}{P_t^{(n)}} = \lim_{n \rightarrow \infty} \beta^{t+n} e^{B_0^n + B_1^n z_t},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$0 < \lim_{n \rightarrow \infty} \frac{P_t^{(n)}}{\beta^n} < \infty.$$

The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n \rightarrow \infty} B_0^n - B_0^{n-1} = \alpha + B_1^\infty(1 - \phi)\theta$, where B_1^∞ is defined implicitly in a second-order equation above. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\alpha - B_1^\infty(1 - \phi)\theta}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the SDF is thus equal to:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = \beta e^{B_1^\infty(z_{t+1} - z_t)} = \beta e^{B_1^\infty[(\phi - 1)(z_t - \theta) - \sigma\sqrt{z_t}u_{t+1}]}$$

As a result, the martingale component of the SDF is then:

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \right)^{-1} = \beta^{-1} e^{-\alpha - \chi z_t - \sqrt{\gamma}z_t u_{t+1}} e^{-B_1^\infty[(\phi - 1)(z_t - \theta) - \sigma\sqrt{z_t}u_{t+1}]} \quad (26)$$

Decomposition (Hansen and Scheinkman (2009)) We now show that the Hansen and Scheinkman (2009) methodology leads to similar results. We guess an eigenfunction ϕ of the form

$$\phi(x) = e^{cx}$$

where c is a constant. Then, the (one-period) eigenfunction problem can be written as

$$E_t[\exp(\alpha + \chi z_t + \sqrt{\gamma}z_t u_{t+1} + cz_{t+1})] = \exp(\beta + cz_t).$$

Expanding and matching coefficients, we get:

$$\begin{aligned} \beta &= -\alpha + c(1 - \phi)\theta \\ \left[\frac{1}{2}\sigma^2 \right] c^2 + [\sigma\sqrt{\gamma} + \phi - 1]c + \left[\frac{1}{2}\gamma - \chi \right] &= 0 \end{aligned}$$

so c solves a quadratic equation. The transitory component of the pricing kernel is by definition:

$$\Lambda_t^{\mathbb{T}} = e^{\beta t - cz_t}$$

The transitory and permanent SDF component are thus:

$$\begin{aligned} \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} &= e^{\beta - c(z_{t+1} - z_t)} = e^{\beta - c[(1 - \phi)(\theta - z_t) - \sigma\sqrt{z_t}u_{t+1}]} = e^{-\alpha + c[(1 - \phi)z_t + \sigma\sqrt{z_t}u_{t+1}]} \\ \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} &= \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \right)^{-1} = e^{-\alpha - \chi z_t - \sqrt{\gamma}z_t u_{t+1}} e^{\alpha - c[(1 - \phi)z_t + \sigma\sqrt{z_t}u_{t+1}]} = e^{-[\chi + c(1 - \phi)]z_t - [\sqrt{\gamma} + c\sigma]\sqrt{z_t}u_{t+1}} \end{aligned}$$

The law of motion of bond prices implies that $c = -B_1^\infty$. If $\chi = -c(1 - \phi)$, then the quadratic equation for c becomes

$$\sigma^2 c^2 + 2\sigma\sqrt{\gamma}c + \gamma = 0$$

with unique solution $c = -\frac{\sqrt{\gamma}}{\sigma}$. Then, the martingale component of the SDF becomes

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = 1$$

so the entirety of the SDF is its transitory component.

Term Premium The expected log excess return is thus given by:

$$E_t[rx_{t+1}^{(n)}] = \left[-\frac{1}{2} (B_1^{n-1})^2 \sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}\right]z_t.$$

The expected log excess return of an infinite maturity bond is then:

$$\begin{aligned} E_t[rx_{t+1}^{(\infty)}] &= \left[-\frac{1}{2} (B_1^\infty)^2 \sigma^2 + \sigma\sqrt{\gamma}B_1^\infty\right]z_t, \\ &= [B_1^\infty(1 - \phi) - \chi + \frac{1}{2}\gamma]z_t. \end{aligned}$$

The $-\frac{1}{2} (B_1^\infty)^2 \sigma^2$ is a Jensen term. The term premium is driven by $\sigma\sqrt{\gamma}B_1^\infty z_t$, where B_1^∞ is defined implicitly in the second order equation $B_1^\infty = \chi - \frac{1}{2}\gamma + B_1^\infty\phi - \frac{1}{2} (B_1^\infty)^2 \sigma^2 + \sigma\sqrt{\gamma}B_1^\infty$.

Model with Country-specific Factors Consider the special case of $B_1^\infty(1 - \phi) = \chi$. In this case, the expected term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\gamma z_t$, which is equal to one-half of the variance of the log stochastic discount factor.

Suppose that the foreign pricing kernel is specified as in Equation (30) with the same parameters. However, the foreign country has its own factor z^* . As a result, the difference between the domestic and foreign log term premia is equal to the log currency risk premium, which is given by $E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(z_t - z_t^*)$. In other words, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma z_t$.

This special case corresponds to the absence of permanent shocks to the SDF: when $B_1^\infty(1 - \phi) = \chi$, the permanent component of the stochastic discount factor is constant. To see this result, let us go back to the implicit definition of B_1^∞ in Equation (26):

$$\begin{aligned} 0 &= \frac{1}{2} (B_1^\infty)^2 \sigma^2 + (1 - \phi - \sigma\sqrt{\gamma})B_1^\infty - \chi + \frac{1}{2}\gamma, \\ 0 &= \frac{1}{2} (B_1^\infty)^2 \sigma^2 - \sigma\sqrt{\gamma}B_1^\infty + \frac{1}{2}\gamma, \\ 0 &= (\sigma B_1^\infty - \sqrt{\gamma})^2. \end{aligned}$$

In this special case, $B_1^\infty = \sqrt{\gamma}/\sigma$. Using this result in Equation (26), the permanent component of the SDF reduces to:

$$\frac{M_{t+1}^{\mathbb{P}}}{M_t^{\mathbb{P}}} = \frac{M_{t+1}}{M_t} \left(\frac{M_{t+1}^{\mathbb{T}}}{M_t^{\mathbb{T}}} \right)^{-1} = \beta^{-1} e^{-\alpha - \chi z_t - \sqrt{\gamma} z_t u_{t+1}} e^{-B_1^\infty [(\phi-1)(z_t - \theta) - \sigma\sqrt{z_t} u_{t+1}]} = \beta^{-1} e^{-\alpha - \chi\theta},$$

which is a constant.

Model with Global Factors We assume that all the shocks are global and that z_t is a global state variable (and thus $\sigma = \sigma^*$, $\phi = \phi^*$, $\theta = \theta^*$). The state variable is referred as “permanent” if it has some impact on the permanent component of the SDF. The difference in term premia between the domestic and foreign bond (once expressed in the same currency) is pinned down by the difference in conditional variances of the permanent components of the SDFs. Therefore the two bonds have the same risk premia when:

$$\sqrt{\gamma} + B_1^\infty \sigma = \sqrt{\gamma^*} + B_1^{\infty*} \sigma$$

Note that B_1^∞ depends on χ and γ , as well as on the global parameters ϕ and σ . The domestic and foreign infinite-maturity bonds have the same risk premia (once expressed in the same currency) when $\gamma = \gamma^*$ and $\chi = \chi^*$, i.e. when the domestic and foreign SDFs react similarly to changes in the global “permanent” state variable and its shocks

C.3 Multi-Factor Vasicek Models

Model Under some conditions, the previous results can be extended to a more k -factor model. The standard k -factor essentially affine model in discrete time generalizes the Vasicek (1977) model to multiple risk factors. The log SDF is given by:

$$-\log M_{t+1} = y_{1,t} + \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + \Lambda_t'\varepsilon_{t+1}$$

To keep the model affine, the law of motion of the risk-free rate and of the market price of risk are:

$$\begin{aligned} y_{1,t} &= \delta_0 + \delta_1'x_t, \\ \Lambda_t &= \Lambda_0 + \Lambda_1x_t, \end{aligned}$$

where the state vector ($x_t \in R^k$) is:

$$x_{t+1} = \Gamma x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma).$$

x_t is a $k \times 1$ vector, and so are ε_{t+1} , δ_1 , Λ_t , and Λ_0 , while Γ , Λ_1 , and Σ are $k \times k$ matrices.²²

We assume that the market price of risk is constant ($\Lambda_1 = \mathbf{0}$), so that we can define orthogonal temporary shocks. We decompose the shocks into two groups: the first $h < k$ shocks affect both the temporary and the permanent SDF components and the last $k - h$ shocks are temporary.²³ The parameters of the temporary shocks satisfy $B_{1k-h}^{\infty'} = (I_{k-h} - \Gamma_{k-h})^{-1}\delta_{1k-h}' = -\Lambda_{0k-h}'$. This ensures that these shocks do not affect the permanent component of the SDF.

Symmetric Model with Global Factor Now we assume that x_t is a global state variable:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2}\Lambda_t^{*'}\Sigma\Lambda_t^* + \Lambda_t^{*'}\varepsilon_{t+1}, \\ y_{1,t} &= \delta_0^* + \delta_1^{*'}x_t, \\ \Lambda_t^* &= \Lambda_0^*, \\ x_{t+1} &= \Gamma x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma). \end{aligned}$$

In a multi-factor Vasicek model with global factors and constant risk prices, long bond uncovered return parity obtains only if countries share the same Λ_h and δ_{1h} , which govern exposure to the permanent, global shocks.

This condition eliminates any differences in permanent risk exposure across countries.²⁴ The nominal exchange rate has no permanent component ($\frac{S_t^P}{S_{t+1}^P} = 1$). From equation (21), the expected log currency excess return is equal to:

$$E_t[r x_{t+1}^{FX}] = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\Lambda_0'\Sigma\Lambda_0 - \frac{1}{2}\Lambda_0^{*'}\Sigma\Lambda_0^*.$$

Non-zero currency risk premia will be only due to variation in the exposure to transitory shocks (Λ_{0k-h}^*).

C.4 Gaussian Dynamic Term Structure Models

Model The k -factor heteroskedastic Gaussian Dynamic Term Structure Model (DTSM) generalizes the CIR model. When market prices of risk are constant, the log SDF is given by:

$$\begin{aligned} -m_{t+1} &= y_{1,t} + \frac{1}{2}\Lambda'V(x_t)\Lambda + \Lambda'V(x_t)^{1/2}\varepsilon_{t+1}, \\ x_{t+1} &= \Gamma x_t + V(x_t)^{1/2}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, I), \\ y_{1,t} &= \delta_0 + \delta_1'x_t, \end{aligned}$$

²²Note that if $k = 1$ and $\Lambda_1 = 0$, we are back to the Vasicek (1977) model with one factor and a constant market price of risk. The Vasicek (1977) model presented in the first section is a special case where $\Lambda_0 = \lambda$, $\delta_0' = \delta$, $\delta_0' = 1$ and $\Gamma = \rho$.

²³A block-diagonal matrix whose blocks are invertible is invertible, and its inverse is a block diagonal matrix (with the inverse of each block on the diagonal). Therefore, if Γ is block-diagonal and $(I - \Gamma)$ is invertible, we can decompose the shocks as described.

²⁴The terms δ_1' and $\delta_{1h}^{*'}$ do not appear in the single-factor Vasicek (1977) model of the first section because that single-factor model assumes $\delta_1 = \delta_{1h}^* = 1$.

where $V(x)$ is a diagonal matrix with entries $V_{ii}(x_t) = \alpha_i + \beta'_i x_t$. To be clear, x_t is a $k \times 1$ vector, and so are ε_{t+1} , Λ , δ_1 , and β_i . But Γ and V are $k \times k$ matrices. A restricted version of the model would impose that β_i is a scalar and $V_{ii}(x_t) = \alpha_i + \beta_i x_{it}$ — this is equivalent to assuming that the price of shock i only depends on the state variable i .

Bond Prices The price of a one period-bond is:

$$P_t^{(1)} = E_t(M_{t+1}) = e^{-\delta_0 - \delta'_1 x_t}.$$

For any maturity n , bond prices are exponentially affine, $P_t^{(n)} = \exp(-B_0^n - B_1^{n'} x_t)$. Note that B_0^n is a scalar, while B_1^n is a $k \times 1$ vector. The one period-bond corresponds to $B_0^1 = \delta_0$, $B_1^1 = \delta'_1$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t(M_{t+1} P_{t+1}^{(n-1)})$, which implies:

$$\begin{aligned} P_t^{(n)} &= E_t \left(\exp \left(-y_{1,t} - \frac{1}{2} \Lambda' V(x_t) \Lambda - \Lambda' V(x_t)^{1/2} \varepsilon_{t+1} - B_0^{n-1} - B_1^{n-1'} x_{t+1} \right) \right) \\ &= \exp(-B_0^n - B_1^{n'} x_t). \end{aligned}$$

This delivers the following difference equations:

$$\begin{aligned} B_0^n &= \delta_0 + B_0^{n-1} - \frac{1}{2} B_1^{n-1'} V(0) B_1^{n-1} - \Lambda' V(0) B_1^{n-1}, \\ B_1^{n'} &= \delta'_1 + B_1^{n-1'} \Gamma - \frac{1}{2} B_1^{n-1'} V_x B_1^{n-1} - \Lambda' V_x B_1^{n-1}, \end{aligned}$$

where V_x denotes all the diagonal slope coefficients β_i of the V matrix.

The CIR model studied in the previous pages is a special case of this model. It imposes that $k = 1$, $\sigma = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma} \sqrt{\gamma}$. Note that the CIR model has no constant term in the square root component of the log SDF, but that does not imply $V(0) = 0$ here as the CIR model assumes that the state variable has a non-zero mean (while it is zero here).

Decomposition (Alvarez and Jermann, 2005) From there, we can define the Alvarez and Jermann (2005) pricing kernel components as for the Vasicek model. The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n \rightarrow \infty} B_0^n - B_0^{n-1} = \delta_0 - \frac{1}{2} B_1^{\infty'} V(0) B_1^\infty - \Lambda'_0 V(0) B_1^\infty$, where $B_1^{\infty'}$ is the solution to the second-order equation above. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\delta_0 + \frac{1}{2} B_1^{\infty'} V(0) B_1^\infty + \Lambda'_0 V(0) B_1^\infty}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the pricing kernel is thus equal to:

$$\frac{\Lambda_{t+1}^\top}{\Lambda_t^\top} = \beta e^{B_1^{\infty'}(x_{t+1} - x_t)} = \beta e^{B_1^{\infty'}(\Gamma - 1)x_t + B_1^{\infty'} V(x_t)^{1/2} \varepsilon_{t+1}}.$$

The martingale component of the pricing kernel is then:

$$\begin{aligned} \frac{\Lambda_{t+1}^\mathbb{P}}{\Lambda_t^\mathbb{P}} &= \frac{\Lambda_{t+1}^\top}{\Lambda_t^\top} \left(\frac{\Lambda_{t+1}^\top}{\Lambda_t^\top} \right)^{-1} = \beta^{-1} e^{-B_1^\infty(\Gamma - 1)x_t - B_1^{\infty'} V(x_t)^{1/2} \varepsilon_{t+1} - y_{1,t} - \frac{1}{2} \Lambda' V(x_t) \Lambda_t - \Lambda' V(x_t)^{1/2} \varepsilon_{t+1}} \\ &= \beta^{-1} e^{-B_1^\infty(\Gamma - 1)x_t - \delta_0 - \delta'_1 x_t - \frac{1}{2} \Lambda' V(x_t) \Lambda - (\Lambda' + B_1^{\infty'}) V(x_t)^{1/2} \varepsilon_{t+1}}. \end{aligned}$$

For the martingale component to be constant, we need that $\Lambda' = -B_1^{\infty'}$ and $B_1^\infty(\Gamma - 1) + \delta'_1 + \frac{1}{2} \Lambda' V_x \Lambda = 0$. Note that the second condition is automatically satisfied if the first one holds: this result comes from the implicit value of $B_1^{\infty'}$ implied by the law of motion of B_1 . As a result, the martingale component is constant as soon as $\Lambda = -B_1^\infty$.

Decomposition (Hansen and Scheinkman (2009)) We guess an eigenfunction ϕ of the form

$$\phi(x) = e^{c'x}$$

where c is a $k \times 1$ vector of constants. Then, the (one-period) eigenfunction problem can be written as

$$E_t \left[\exp(-\delta_0 - \delta'_1 x_t - \frac{1}{2} \Lambda' V(x_t) \Lambda - \Lambda' V^{1/2}(x_t) \varepsilon_{t+1} + c x_{t+1}) \right] = \exp(\beta + c x_t)$$

Expanding and matching coefficients, we get:

$$\begin{aligned}\beta &= -\delta_0 - \frac{1}{2}\Lambda'V(0)\Lambda + \frac{1}{2}(c - \Lambda)'V(0)(c - \Lambda) \\ 0 &= c'(\Gamma - I) - \delta'_1 + \sum_{i=1}^k c_i(c_i - 2\Lambda_i)\mu'_i.\end{aligned}$$

The transitory component of the pricing kernel is by definition:

$$\Lambda_t^\mathbb{T} = e^{\beta t - c'x_t}$$

The transitory and permanent SDF component are thus:

$$\begin{aligned}\frac{\Lambda_{t+1}^\mathbb{T}}{\Lambda_t^\mathbb{T}} &= e^{\beta - c'(x_{t+1} - x_t)} = e^{\beta - c'(\Gamma - I)x_t - c'V^{1/2}(x_t)\varepsilon_{t+1}} \\ \frac{\Lambda_{t+1}^\mathbb{P}}{\Lambda_t^\mathbb{P}} &= \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\Lambda_{t+1}^\mathbb{T}}{\Lambda_t^\mathbb{T}} \right)^{-1} \\ &= e^{-\delta_0 - \delta'_1 x_t - \frac{1}{2}\Lambda'V(x_t)\Lambda - \Lambda'V^{1/2}(x_t)\varepsilon_{t+1}} e^{-\beta + c'(\Gamma - I)x_t + c'V^{1/2}(x_t)\varepsilon_{t+1}} \\ &= e^{-\delta_0 - \beta + [c'(\Gamma - I) - \delta'_1]x_t - \frac{1}{2}\Lambda'V(x_t)\Lambda + (c - \Lambda)'V^{1/2}(x_t)\varepsilon_{t+1}}\end{aligned}$$

If $\Lambda = c$, then the equations for β and c become:

$$\begin{aligned}\beta &= -\delta_0 - \frac{1}{2}c'V(0)c \\ 0 &= c'(\Gamma - I) - \delta'_1 - \sum_{i=1}^k c_i^2\mu'_i\end{aligned}$$

The martingale component of the SDF is then

$$\frac{\Lambda_{t+1}^\mathbb{P}}{\Lambda_t^\mathbb{P}} = e^{-\delta_0 - \beta + [c'(\Gamma - I) - \delta'_1]x_t - \frac{1}{2}c'V(x_t)c} = 1$$

The entirety of the SDF is described by its transitory component in this case.

Term Premium The expected log holding period excess return is:

$$E_t[rx_{t+1}^{(n)}] = -\delta_0 + (-B_1^{n-1}\Gamma + B_1^{n'} - \delta'_1)x_t.$$

The term premium on an infinite-maturity bond is therefore:

$$E_t[rx_{t+1}^{(\infty)}] = -\delta_0 + ((1 - \Gamma)B_1^{\infty'} - \delta'_1)x_t.$$

The expected log currency excess return is equal to:

$$E_t[-\Delta s_{t+1}] + y_t^* - y_t = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\Lambda'V(x_t)\Lambda - \frac{1}{2}\Lambda^{*'}V(x_t^*)\Lambda^*.$$

We assume that all the shocks are global and that x_t is a global state variable ($\Gamma = \Gamma^*$ and $V = V^*$, no country-specific parameters in the V matrix— cross-country differences will appear in the vectors Λ). Let us decompose the shocks into two groups: the first $h < k$ shocks affect both the temporary and the permanent SDF components and the last $k - h$ shocks are temporary. Temporary shocks are such that $\Lambda_{k-h} = -B_{1,k-h}^\infty$ (i.e., they do not affect the value of the permanent component of the SDF).

The risk premia on the domestic and foreign infinite-maturity bonds (once expressed in the same currency) will be the same provided that the entropy of the domestic and foreign permanent components is the same:

$$\begin{aligned}(\Lambda'_h + B_{1h}^{\infty'})V(0)(\Lambda_h + B_{1h}^\infty) &= (\Lambda_h^{*'} + B_{1h}^{*\infty'})V(0)(\Lambda_h^* + B_{1h}^{\infty*}), \\ (\Lambda'_h + B_{1h}^{\infty'})V_x(\Lambda_h + B_{1h}^\infty) &= (\Lambda_h^{*'} + B_{1h}^{*\infty'})V_x(\Lambda_h^* + B_{1h}^{\infty*}).\end{aligned}$$

To compare these conditions to the results obtained in the one-factor CIR model, recall that $\sigma^{CIR} = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma^{CIR}}\sqrt{\gamma^{CIR}}$. Differences in Λ_h in the k -factor model are equivalent to differences in γ in the CIR model: in both cases, they correspond to

different loadings of the log SDF on the “permanent” shocks. As in the CIT model, differences in term premia can also come from differences in the sensitivity of infinite-maturity bond prices to the global “permanent” state variable ($B_{1h}^{\infty'}$), which can be traced back to differences in the sensitivity of the risk-free rate to the “permanent” state variable (i.e., different δ_1 parameters).

Special case Let us start with the special case of no permanent innovations: $h = 0$, the martingale component is constant. Two conditions need to be satisfied for the martingale component to be constant: $\Lambda' = -B_{1h}^{\infty'}$ and $B_{1h}^{\infty}(\Gamma - 1) + \delta_1' + \frac{1}{2}\Lambda'V_x\Lambda = 0$. The second condition imposes that the cumulative impact on the pricing kernel of an innovation today given by $(\delta_1' + \frac{1}{2}\Lambda'V_x\Lambda)(1 - \Gamma)^{-1}$ equals the instantaneous impact of the innovation on the long bond price. The second condition is automatically satisfied if the first one holds, as can be verified from the implicit value of $B_{1h}^{\infty'}$ implied by the law of motion of B_1 . As a result, the martingale component is constant as soon as $\Lambda = -B_{1h}^{\infty}$.

As implied by Equation (22), the term premium on an infinite-maturity zero coupon bond is:

$$E_t[rx_{t+1}^{(\infty)}] = -\delta_0 + ((1 - \Gamma)B_{1h}^{\infty'} - \delta_1')x_t. \quad (27)$$

In the absence of permanent shocks, when $\Lambda = -B_{1h}^{\infty}$, this log bond risk premium equals half of the stochastic discount factor variance $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\Lambda'V(x_t)\Lambda$; it attains the upper bound on log risk premia. Consistent with the result in Equation (21), the expected log currency excess return is equal to:

$$E_t[rx_{t+1}^{FX}] = \frac{1}{2}\Lambda'V(x_t)\Lambda - \frac{1}{2}\Lambda^{*'}V(x_t)\Lambda^*. \quad (28)$$

Differences in the market prices of risk Λ imply non-zero currency risk premia. Adding the previous two expressions in Equations (27) and (28), we obtain the foreign bond risk premium in dollars. The foreign bond risk premium in dollars equals the domestic bond premium in the absence of permanent shocks: $E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\Lambda'V(x_t)\Lambda$.

General case In general, there is a spread between dollar returns on domestic and foreign bonds. We describe a general condition for long-run uncovered return parity in the presence of permanent shocks.

Result 4. *In a GDTSM with global factors, the long bond uncovered return parity condition holds only if the countries’ SDFs share the parameters $\Lambda_h = \Lambda_h^*$ and $\delta_{1h} = \delta_{1h}^*$, which govern exposure to the permanent global shocks.*

The log risk premia on the domestic and foreign infinite-maturity bonds (once expressed in the same currency) are identical provided that the entropies of the domestic and foreign permanent components are the same:

$$\begin{aligned} (\Lambda_h' + B_{1h}^{\infty'})V(0)(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*'} + B_{1h}^{*\infty'})V(0)(\Lambda_h^* + B_{1h}^{*\infty}), \\ (\Lambda_h' + B_{1h}^{\infty'})V_x(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*'} + B_{1h}^{*\infty'})V_x(\Lambda_h^* + B_{1h}^{*\infty}). \end{aligned}$$

These conditions are satisfied if that these countries share $\Lambda_h = \Lambda_h^*$ and $\delta_{1h} = \delta_{1h}^*$ which govern exposure to the global shocks. In this case, the expected log currency excess return is driven entirely by differences between the exposures to transitory shocks: Λ_{k-h} and Λ_{k-h}^* . If there are only permanent shocks ($h = k$), then the currency risk premium is zero.²⁵

²⁵To compare these conditions to the results obtained in the CIR model, recall that we have constrained the parameters in the CIR model such that: $\sigma^{CIR} = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma^{CIR}}\sqrt{\gamma^{CIR}}$. Differences in Λ_h in the k -factor model are equivalent to differences in γ in the CIR model: in both cases, they correspond to different loadings of the log SDF on the “permanent” shocks. Differences in term premia can also come from differences in the sensitivity of the risk-free rate to the permanent state variable (i.e., different δ_1 parameters). These correspond to differences in χ in the CIR model.

D Market Incompleteness

We want to verify whether our results are robust to market incompleteness. We consider a world in which foreign investors can only take positions in forward markets, but they cannot trade other foreign assets directly. This type of incompleteness introduces a wedge η_{t+1} between the spot exchange rates and the pricing kernels:

$$\frac{S_{t+1}}{S_t} = \frac{M_{t+1}}{M_{t+1}^*} e^{-\eta_{t+1}} \quad (29)$$

Obviously, we need to make sure that the Euler equations for the domestic and the foreign risk-free are satisfied:

$$\begin{aligned} E_t(M_{t+1}^*) &= E_t\left(M_{t+1} \frac{S_t}{S_{t+1}}\right) = E_t(M_{t+1}^* e^{\eta_{t+1}}) = 1/R_t^{f,*}, \\ E_t(M_{t+1}) &= E_t\left(M_{t+1}^* \frac{S_{t+1}}{S_t}\right) = E_t(M_{t+1} e^{-\eta_{t+1}}) = 1/R_t^f, \end{aligned}$$

where we have used the relation in Equation (29) in the last equality. We implement these restrictions in a two-country CIR model defined by the law of motion of the home and foreign SDFs:

$$\begin{aligned} -\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma} z_t u_{t+1}, \\ z_{t+1} &= (1 - \phi)\theta + \phi z_t - \sigma \sqrt{z_t} u_{t+1}, \\ -\log M_{t+1}^* &= \alpha + \chi z_t^* + \sqrt{\gamma^*} z_t^* u_{t+1}^*, \\ z_{t+1}^* &= (1 - \phi)\theta + \phi z_t^* - \sigma \sqrt{z_t^*} u_{t+1}^*, \end{aligned}$$

In this CIR model, Lustig and Verdelhan (2015) derive restrictions on the wedges that need to be satisfied in order to rule out arbitrage opportunities in forward markets.

Proposition. *In the CIR model, the incomplete market wedge η_{t+1} (such that $\Delta s_{t+1} = m_{t+1} - m_{t+1}^* - \eta_{t+1}$) follows:*

$$\eta_{t+1} = \psi z_t + \psi^* z_t^* - \sqrt{(\gamma - \lambda) z_t} u_{t+1} + \sqrt{(\gamma^* - \lambda^*) z_t^*} u_{t+1}^* + \sqrt{(\lambda - \kappa) z_t} \epsilon_{t+1} + \sqrt{(\lambda^* - \kappa^*) z_t^*} \epsilon_{t+1}^*, \quad (30)$$

where the shocks ϵ_{t+1} and ϵ_{t+1}^* are normally distributed as $N(0, 1)$ and orthogonal to the SDF shocks u_{t+1} and u_{t+1}^* . The law of motion of the wedge is determined by two parameters λ and λ^* that satisfy $\lambda \leq \gamma$ and $\lambda^* \leq \gamma^*$. Once these two parameters are fixed, they determine the values of the remaining parameters $\kappa \leq \lambda$, $\kappa^* \leq \lambda^*$, ψ , and ψ^* :

$$\kappa^* = \gamma^* - \sqrt{\gamma^*} \sqrt{\gamma^* - \lambda^*}, \quad (31)$$

$$\psi^* = \frac{1}{2}(\gamma^* - \kappa^*), \quad (32)$$

$$\kappa = \gamma - \sqrt{\gamma} \sqrt{\gamma - \lambda}, \quad (33)$$

$$\psi = -\frac{1}{2}(\gamma - \kappa). \quad (34)$$

As noted in Proposition 5, when markets are complete and the long-term bond returns, once converted in the same currency, are the same, then the permanent components of the home and foreign SDFs are the same, thus implying that nominal exchange rates are stationary in levels. When markets are incomplete, however, long-term bond returns, expressed in the same currency, could be the same, without implying that the permanent components of the home and foreign SDFs are identical. When markets are incomplete, the ratio of foreign to domestic bond returns, expressed in the same currency is equal to:

$$R_{t+1}^{(\infty),*} \frac{S_t}{S_{t+1}} / R_{t+1}^{(\infty)} = e^{\eta_{t+1}} M_{t+1}^{\mathbb{P}^*} / M_{t+1}^{\mathbb{P}}. \quad (35)$$

Let us assume that the long-term bond return parity condition holds at each date, but that the ratio of the permanent components of the SDFs is equal to the incomplete market wedge:

$$M_{t+1}^{\mathbb{P}} / M_{t+1}^{\mathbb{P}^*} = e^{\eta_{t+1}}. \quad (36)$$

In this case, the long-term bond return parity condition holds but the nominal exchange rate is not necessarily stationary in level. This counterexample to our main result, however, holds only in a knife-edge case. Recall that in the CIR model, the permanent

components of the home and foreign SDFs are:

$$\begin{aligned} M_{t+1}^{\mathbb{P}} &= \delta^{-1} e^{-\alpha - \chi z_t - \sqrt{\gamma z_t} u_{t+1}} e^{-B_1^\infty [(\phi-1)(z_t - \theta) - \sigma \sqrt{z_t} u_{t+1}]}, \\ M_{t+1}^{\mathbb{P}^*} &= \delta^{-1} e^{-\alpha - \chi z_t^* - \sqrt{\gamma^* z_t^*} u_{t+1}^*} e^{-B_1^{\infty*} [(\phi-1)(z_t^* - \theta) - \sigma \sqrt{z_t^*} u_{t+1}^*]}. \end{aligned}$$

The ratio of the home and foreign permanent components is thus:

$$M_{t+1}^{\mathbb{P}}/M_{t+1}^{\mathbb{P}^*} = e^{-\chi(z_t - z_t^*) - \sqrt{\gamma z_t} u_{t+1} + \sqrt{\gamma^* z_t^*} u_{t+1}^*} e^{-B_1^\infty [(\phi-1)(z_t - \theta) - \sigma \sqrt{z_t} u_{t+1}] + B_1^{\infty*} [(\phi-1)(z_t^* - \theta) + \sigma \sqrt{z_t^*} u_{t+1}^*]}$$

As noted in the Proposition above in Equation (30), the law of motion of the incomplete market wedge is:

$$\eta_{t+1} = \psi z_t + \psi^* z_t^* - \sqrt{(\gamma - \lambda) z_t} u_{t+1} + \sqrt{(\gamma^* - \lambda^*) z_t^*} u_{t+1}^* + \sqrt{(\lambda - \kappa) z_t} \epsilon_{t+1} + \sqrt{(\lambda^* - \kappa^*) z_t^*} \epsilon_{t+1}^*,$$

The ratio of the home to foreign permanent components of the SDFs can be equal to the incomplete market wedge under some specific conditions. It must be that the wedge does not load on the orthogonal shocks ϵ_{t+1} and ϵ_{t+1}^* because those shocks do not affect the permanent components of the SDFs. This implies that $\lambda = \kappa$ and $\lambda^* = \kappa^*$. In order to satisfy the conditions in Equations (31) and (33), such constraints on the coefficients imply that $\kappa = \kappa^* = 0$.²⁶ Gathering these different conditions, it must be that the law of motion of the incomplete market wedge is simply:

$$\eta_{t+1} = -\frac{1}{2} \gamma z_t + \frac{1}{2} \gamma z_t^* - \sqrt{\gamma z_t} u_{t+1} + \sqrt{\gamma^* z_t^*} u_{t+1}^*.$$

In the class of CIR models, this is the only incomplete market wedge that can potentially be a counterexample to our main claim. In order for Equation (36) to hold (and thus exchange rates to be non stationary in levels even when the long-term bond return parity condition holds), the wedge must take this precise form and some additional restrictions on the parameters of the CIR model must hold. In the CIR example above, it must be that $\gamma/2 = \chi$ and $B_1^\infty = B_1^{\infty*} = 0$. In this case, the exchange rate, $\Delta s_{t+1} = m_{t+1} - m_{t+1}^* - \eta_{t+1}$, is actually constant and the term premium is zero.²⁷ To sum up, one can certainly write an incomplete market model where exchange rates are non stationary in levels even if the long-term bond return parity condition holds, but this will be a knife-edge case in a large class of models.

²⁶Note that the other solutions to Equations (31) and (33) imply that $\gamma = \kappa$ and $\gamma^* = \kappa^*$, which would imply that $\psi = 0$ and $\psi^* = 0$, thus violating Equation (36) because the ratio of permanent SDFs does load on the state variables z_t and z_t^* , whereas the wedge would not in this case.

²⁷Note that the same result occurs even when countries differ in their impatience parameter, $\delta \neq \delta^*$. In that case, the ratio of the home and foreign permanent components becomes:

$$M_{t+1}^{\mathbb{P}}/M_{t+1}^{\mathbb{P}^*} = \delta^* \delta^{-1} e^{-\chi(z_t - z_t^*) - \sqrt{\gamma z_t} u_{t+1} + \sqrt{\gamma^* z_t^*} u_{t+1}^*} e^{-B_1^\infty [(\phi-1)(z_t - \theta) - \sigma \sqrt{z_t} u_{t+1}] + B_1^{\infty*} [(\phi-1)(z_t^* - \theta) + \sigma \sqrt{z_t^*} u_{t+1}^*]}$$

In order for Equation (36) to hold, it then must be that $\delta^* = 0$, $\chi = \gamma/2$, $B_1^\infty = 0$ and $B_1^{\infty*} = 0$, which again imply that the exchange rate is constant and the term premium is zero.